**Practice Problem Set 4 - With Solutions**

**Question 1 (1 point)**

**What is Lenz's Law? To which basic principle of physics is it most closely related?**

1) Lenz's law = The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.

It is closely related to conservation of energy.

**Question 2 (3 points)**

**A circular coil of wire with 350 turns and a radius of 7.5 cm is placed horizontally on a table. A uniform magnetic field pointing directly up is slowly turned on, such that the strength of the magnetic field can be expressed as a function of time as: $B(t) = 0.02(T/s^2) \times t^2$. What is the total EMF in the coil as a function of time? In which direction does the current flow?**

2) EMF = (-N) * (pi * r ^ 2 ) * (d/dt B) = -350 * pi * (0.075 m)^2 * 2 * 0.020 T * t
 = - .25 t (Tm^2/ s^2)
 = -.25 t V/s

Clockwise - looking from the top

**Question 3 (3 points)**

**A metal bar with a resistance of 30 Ω is rotated around its center in a magnetic field of strength 0.5 T which is oriented perpendicularly to the plane of the bar's rotation. If the bar makes 3 full rotations per second, what is the electrical power dissipated in the resistor?**

3) P = E ^ 2 (<-- EMF) / R
 dE = Bvdr
 E = Intergal of Bvdr
 E = B * Intergal of vdr
 = Bw Intrgal of rdr going from -l/2 to l/2
 = Bw (1/2 r^2) with limits from -l/2 to l/2

For the First Half of the Rod:
 E (EMF) = Bw (1/2 r^2) with limits zero to l / 2
 = Bw * 1/2 * (l/2)^2 - 0
 = 1/8 B * w * l^2
 = .375 * pi * l^2 (Units: Tm^2/s)

For the Second Half of the Rod:
 E (EMF) = Bw (1/2 r^2) with limits zero to l / 2

= Bw * 1/2 * (l/2)^2 - 0
= 1/8 B * w * l^2
= .375 * pi * l^2 (Units: Tm^2/s)

Electric Power = P = first half power + second half power
= EMF^2/ R + EMF^2/R
= [(375 * pi * l^2 (Units: Tm^2/s))^2 / (30 Ω)] + (375 * pi * l^2 (Units: Tm^2/s))^2 / (30 Ω)]
= .093 l^4 w/m^4

**Question 4 (3 points)**

**Use Gauss' Law and Ampere's Law to find both the capacitance per unit length and the inductance per unit length of a coaxial cable with an outer radius of 4.5 mm and an inner radius of 1.5 mm. Assume the space between the two conductors is filled with air. An AC generator with a variable frequency is connected across a 45 cm piece of the coaxial cable. If the cable has minimal resistance, at what frequency will the cable resonate?**

- Solving for capacitance per unit length:

$$\oiint \vec{E} \cdot d\vec{A}$$

$$\oiint |E||dA| = \frac{q}{\varepsilon_0}$$

$$|E| = \frac{q}{\varepsilon_0 \oint dA} = \frac{q}{\varepsilon_0} = \frac{q}{\varepsilon_0 2\pi r l}$$

$$V = \int \vec{E} \cdot d\vec{s} = \int_{r_1}^{r_2} \vec{E} \cdot d\vec{r} = \int_{r_1}^{r_2} |E| dr = \int_{r_1}^{r_2} \frac{q\,dr}{\varepsilon_0 2\pi r l} = \frac{q}{\varepsilon_0 2\pi l} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{q}{\varepsilon_0 2\pi l}\left(ln\left(\frac{r_2}{r_1}\right)\right)$$

$$C = \frac{Q}{r} = \frac{q}{\frac{q}{\varepsilon_0 2\pi r l} ln\left(\frac{r_2}{r_1}\right)} = \frac{\varepsilon_0 2\pi r l}{ln\left(\frac{r_2}{r_1}\right)}$$

$$\frac{C}{l} = \frac{\varepsilon_0 2\pi r}{ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi\left(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}\right)}{ln(3)} = 50.6\ \frac{pF}{m}\ or\ 5.06 \times 10^{-11} \frac{F}{m}$$

- Solving for Inductance  per unit length:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$\oint |B||ds| = \mu_0 I$$

$$|B| = \frac{\mu_0 I}{2\pi r}$$

$$\frac{L}{l} = \frac{\Phi_B}{Il} = \frac{\oiint \vec{E} \cdot d\vec{A}}{Il} = \frac{\oiint |E||dA|}{Il} = \frac{\oiint |B|ldr}{Il} = \frac{\oiint |B|dr}{I} = \frac{\oiint \frac{\mu_0 I}{2\pi r} dr}{I}$$

$$= \frac{\mu_0}{2\pi} \int \frac{dr}{r} = \frac{\mu_0}{2\pi} ln\left(\frac{r_2}{r_1}\right) = \frac{4\pi \times 10^{-7} \frac{T \cdot m}{A}}{2\pi} ln(3) = 2.2 \times 10^{-7} \frac{H}{m}$$

Solving for frequency gives:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{l^2 LC}} = \frac{1}{\sqrt{(.45)^2\left(2.2 \times 10^{-7} \frac{H}{m}\right)\left(5.06 \times 10^{-11} \frac{F}{m}\right)}} = 670 MHz$$