Section 5.1: Random Variables

1 Two Examples

Definition 1 *A **random variable** is a function that assigns a numerical value to each outcome in a sample space.*

Example 2 *Consider the experiment of rolling a pair of dice and summing the faces. The random variable X assigns to each roll its sum. The following table indicates the probabilities for each value in X..*

Sum	2	3	4	5	6	7	8	9	10	11	12
Prob	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Example 3 *Let Y be the random variable that assigns to each human being, their height in inches. Given the uncountably infinite number of possible heights, it is not possible to define Y based on a table.*

There are two types of random variables, discrete and continuous, based on the data types assigned to values. The above two examples illustrate the different ways a random variable must be defined. Accordingly, different techniques will be needed to work with the two different types of random variables.

Problem 4 *Consider the experiment of rolling a three sided-die and a 4-sided die and summing the faces. Construct a random variable X and the table that assigns to each sum the probability of attaining that sum.*

2 Exercises

Kokoska 3rd edition Section 5.1: 5.10, 5.11, 5.15-5.17

Section 5.2: Probability Distributions for Discrete Random Variables

Definition 5 *A **probability distribution** for a discrete random variable X is a function whose domain is all possible values of X and assigns to each $x \in X$ the probability that x occurs. Note that the sum of all probabilities in a distribution must be 1.*

Problem 6 *Is the following function a probability distribution? Explain.*

X	1	2	3
$P(X)$	0.5	0.6	-0.1

Example 7 *Consider the following probability distribution.*

X	1	2	3	4	5	6
$P(X)$	0.3	0.2	0.1	0.1		0.2

1. $P(X = 3) =$
2. $P(X = 4) =$
3. $P(X = 5) =$
4. $P(X \leq 3) =$
5. $P(X < 3) =$
6. $P(2 \leq X \leq 4) =$
7. $P(X > 5) =$

Problem 8 *A \$2 lottery ticket offers four chances to win different amounts of money as indicated by the following probability distribution model.*

Prize	\$0	\$0.5	\$1	\$5	\$50
Prob	$\frac{71}{100}$	$\frac{15}{100}$	$\frac{10}{100}$	$\frac{3}{100}$	$\frac{1}{100}$

1. What is the probability that a ticket wins nothing?

2. What is the probability that a ticket wins money but is a losing proposition overall?

3. Two people buy one ticket each. What is the probability that they both win nothing?

4. Two people buy one ticket each. What is the probability that they both win money?

5. Ten people buy one ticket each. What is the probability that at least one person wins money?

6. Five hundred people buy one ticket each. What is the probability that at least one person wins the $50 prize?

Problem 9 *You draw a card from a deck. If you get a club you get $5. If you get an Ace you get $10. For all other cards you receive nothing. Let X be the random variable of money won. Create a probability distribution model for this game.*

Problem 10 *You draw a card from a deck. If you get a club you get nothing. If you get red card you get $10. If you get a spade you get $15 and get to select another card (without replacement). If the second is another spade you receive an additional $20. You receive nothing for any non-spade card. Create a probability model for this game.*

3 Exercises

1. Kokoska 3rd edition Section 5.2: 5.30-5.33, 5.36, 5.37, 5.46 (let several=5)