Section 5.4: The Binomial Probability Distribution

The normal distribution is one that occurs frequently in statistics. Others exist as well. One very common discrete distribution is the Binomial Probability Distribution.

Definition 1 *An experiment consisting of repeated trials has a* ***Binomial Probability Distribution*** *if and only if*

1. *there are only two outcomes for each trial (success or failure) and the probability p of success is fixed;*
2. *there are a fixed number of trials n;*
3. *the trials are independent.*

Exercise 2 *An experiment consists of flipping a fair coin 10 times and counting the number of tails. Does this experiment have a binomial probability distribution?*

Exercise 3 *A multiple choice test contains 20 questions. Each question has four or five choices for the correct answer. Only one of the choices is correct. With random guessing, does this test have a binomial probability distribution?*

Exercise 4 *A multiple choice test contains 20 questions. Each question has five choices for the correct answer. Only one of the choices is correct. When a student trying their best, does this test have a binomial probability distribution?*

Exercise 5 *A multiple choice test contains 20 questions. Each question has five choices for the correct answer. Only one of the choices is correct. With random guessing, does this test have a binomial probability distribution?*

Exercise 6 *It is known that 30% of all students at the University of Knowhere live off campus. An experiment consists of randomly selecting students until a commuter student is found. Does this experiment have a binomial probability distribution?*

Exercise 7 *An experiment consists of asking 20 students what size water they prefer: small, medium or large. All choices are equally likely. Does this experiment have a binomial probability distribution?*

What does knowing that an experiment has a binomial probability distribution buy us? Quite a lot actually.

Theorem 8 *If an experiment has a binomial probability distribution with* n *trials and probability of success* p *then*

1. *the probability of exactly* k *successes is* $p(k) = \binom{n}{k} p^k (1-p)^{n-k}$*;*

2. *the mean or expected value is* $\mu = np$*;*

3. *and the standard deviation is* $\sigma = \sqrt{np(1-p)}$*.*

Remark 9 *On the TI-83/84 series of calculators,* ***binompdf***(n, p, k) *computes the probability of exactly* k *successes. The command* ***binomcdf***(n, p, k) *computes the probability of* 0 *or* 1 *or* 2 *or...or* k *successes.*

Remark 10 *In R Studio,* ***dbinom***(k, n, p) *computes the probability of exactly* k *successes. The command* ***pbinom***(k, n, p) *computes the probability of* 0 *or* 1 *or* 2 *or...or* k *successes.*

Example 11 *An experiment consists of flipping a fair coin 10 times and counting the number of tails. Find the mean and standard deviation for this binomial probability distribution.*

Since $n = 10$ and $p = \frac{1}{2}$, $\mu = 10 * \frac{1}{2} = 5$ and $\sigma = \sqrt{10 * \frac{1}{2} * (1 - \frac{1}{2})} = 1.\,581\,1$.

Example 12 *An experiment consists of flipping a fair coin 10 times and counting the number of tails. What is the probability of observing exactly five tails?*

Since $n = 10$, $p = \frac{1}{2}$ and $k = 5$, the probability of observing exactly 5 tails is $p(k=5) = \binom{10}{5} * (\frac{1}{2})^5 \left(1 - \frac{1}{2}\right)^{(10-5)} = 0.246\,09$

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> dbinom(5,10,.5)
[1] 0.2460938
>
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Example 13 *An experiment consists of flipping a fair coin 10 times and counting the number of tails. What is the probability of observing at most three tails?*

Here $n = 10$ and $p = \frac{1}{2}$ but $k = 0, 1, 2$, or 3. These are disjoint cases so we use the addition rule four times.

$p(k \le 3) = \binom{10}{0} * (\frac{1}{2})^0 \left(1 - \frac{1}{2}\right)^{(10-0)} + \binom{10}{1} * (\frac{1}{2})^1 \left(1 - \frac{1}{2}\right)^{(10-1)} + \binom{10}{2} * (\frac{1}{2})^2 \left(1 - \frac{1}{2}\right)^{(10-2)} + \binom{10}{3} * (\frac{1}{2})^3 \left(1 - \frac{1}{2}\right)^{(10-3)} = 0.171\,88$

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> pbinom(3,10,.5)
[1] 0.171875
>
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Example 14 *An experiment consists of flipping a fair coin 10 times and counting the number of tails. What is the probability of observing at least nine tails?*

Here $n = 10$ and $p = \frac{1}{2}$ but $k = 9$ or 10.

$p(k \geq 9) = \binom{10}{9} * (\frac{1}{2})^9 \left(1 - \frac{1}{2}\right)^{(10-9)} + \binom{10}{10} * (\frac{1}{2})^{10} \left(1 - \frac{1}{2}\right)^{(10-10)} = 1.0742 \times 10^{-2}$.

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> 1-pbinom(8,10,.5)
[1] 0.01074219
>
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Example 15 *An experiment consists of flipping a fair coin 10 times and counting the number of tails. What is the probability the number of tails falls within two standard deviations of the mean?*

Since $5 \pm 2 * 1.58811$ yields the interval 1.8238 to 8.1762, the probability is $\binom{10}{2} * (\frac{1}{2})^2 \left(1 - \frac{1}{2}\right)^{(10-2)} + \binom{10}{3} * (\frac{1}{2})^3 \left(1 - \frac{1}{2}\right)^{(10-3)} + \binom{10}{4} * (\frac{1}{2})^4 \left(1 - \frac{1}{2}\right)^{(10-4)}$ $+\binom{10}{5} * (\frac{1}{2})^5 \left(1 - \frac{1}{2}\right)^{(10-5)} + \binom{10}{6} * (\frac{1}{2})^6 \left(1 - \frac{1}{2}\right)^{(10-6)} + \binom{10}{7} * (\frac{1}{2})^7 \left(1 - \frac{1}{2}\right)^{(10-7)}$ $+\binom{10}{8} * (\frac{1}{2})^8 \left(1 - \frac{1}{2}\right)^{(10-8)} = 0.97852$.

Problem 16 *Recall that Shaquille O'Neal's lifetime free throw percentage is* 0.527*. Assume that free throw attempts are independent of one another. A teams employs the "Hack-a-Shaq" strategy. In a three-game series, Shaq is fouled 22 times all on a two-point shot.*

1. *Find the mean and standard deviation for the number of made free throws by Shaq.*
2. *Compute the probability that Shaq makes at least 20 free throws.*
3. *Would it be unusual for Shaq to make at least 30 free throws?*
4. *What number of free throws must Shaq make in order to be more than three standard deviations above average?*

Problem 17 *In 2017, 71% of all full-time KSU undergraduates received some type of need-based financial aid (https://www.usnews.com/best-colleges/kennesaw-state-university-1577). Twenty students are selected at random.*

1. *Find the mean and standard deviation for the number of students who received need-based financial aid.*
2. *Compute the probability that exactly fourteen of the twenty students received need-based financial aid.*

3. *Compute the probability that at least fifteen of the twenty students received need-based financial aid.*

4. *Suppose only two of the twenty KSU students on the intramural water-polo team received need-based financial aid. What does that suggest about water-polo.*

1 Exercises

1. Kokoska 3rd edition Section 5.4: 5.85-5.87, 5.89-5.93, 5.96, 5.97, 5.99, 5.103, 5.105, 5.106, 5.110, 5.114