Section 5.5: Other Discrete Random Variables

1 The Geometric Distribution

The necessary properties of a geometric probability distribution are similar to those of the binomial probability distribution.

Definition 1 *An experiment consisting of repeated trials has a* ***Geometric Probability Distribution*** *if and only if*

1. *there are only two outcomes for each trial (success or failure) and the probability p of success is fixed;*
2. *the trials are independent.*

The target of the geometric distribution is the number of trials until the first success. Knowing an experiment has a geometric distribution provides the following formulas.

Theorem 2 *If an experiment has a geometric probability distribution with fixed probability of success p then*

1. *the probability that the first success occurs on trial k is $p(k) = p\,(1-p)^{k-1}$;*
2. *the mean or expected value number of trials until the first success is $\mu = \frac{1}{p}$;*
3. *and the standard deviation is $\sigma = \frac{\sqrt{(1-p)}}{p}$.*

Remark 3 *On the TI-83/84 series of calculators,* ***geometpdf***(p, k) *computes the probability of requiring k trials for the initial success. The command* ***geometcdf***(p, k) *computes the probability of requiring 1 or 2 or...or k trials for the initial success.*

Remark 4 *In R Studio,* ***dgeom***$(k-1, p)$ *computes the probability of requiring k trials for the initial success. The command* ***pgeom***$(k-1, p)$ *computes the probability of requiring 1 or 2 or...or k trials for the initial success. Basically, R Studio wants the number of failures before the first success as input.*

Example 5 *Recall that Shaquille O'Neal's lifetime free throw percentage is 0.527. Assume that free throw attempts are independent of one another.*

1. Find the mean and standard deviation for the number of attempts needed before making a free throw.

   $\mu = \frac{1}{p} = \frac{1}{0.527} = 1.8975$ and $\sigma = \frac{\sqrt{(1-0.527)}}{0.527} = 1.305$

```
[1] 0.8941762
> 1/.527
[1] 1.897533
> sqrt(1-.527)/.527
[1] 1.305028
```

2. What is the probability Shaq makes his first free throw on his fourth attempt?
   $p(4) = 0.527\,(1-0.527)^{4-1} = 5.\,576\,9 \times 10^{-2}$
   ```
   > dgeom(3,.527)
   [1] 0.05576915
   >
   ```

3. What is the probability it takes Shaq no more than three attempts before he makes his first free throw?
   $p(x \leq 3) = p(1)+p(2)+p(3) = 0.527\,(1-0.527)^{1-1}+0.527\,(1-0.527)^{2-1}+ 0.527\,(1-0.527)^{3-1} = 0.894\,18$
   ```
   > pgeom(2,.527)
   [1] 0.8941762
   >
   ```

4. What is the probability Shaq makes his first free throw on his fourth, fifth or sixth attempt?
   $p(4)+p(5)+p(6) = 0.527\,(1-0.527)^{4-1}+0.527\,(1-0.527)^{5-1}+0.527\,(1-0.527)^{6-1} = 9.\,462\,5 \times 10^{-2}$
   ```
   > dgeom(3,.527)+dgeom(4,.527)+dgeom(5,.527)
   [1] 0.09462514
   >
   ```

5. What is the probability it takes Shaq at least three attempts before he makes his first free throw?
   $p(x \geq 3) = 1-p(1)+p(2) = 1-(0.527\,(1-0.527)^{1-1}+0.527\,(1-0.527)^{2-1}) = 0.223\,73$
   ```
   > 1-(dgeom(0,.527)+dgeom(1,.527))
   [1] 0.223729
   ```

Problem 6 *John intends to roll a pair of dice until he gets a sum of 12.*

1. *How many times do you expect John to roll the dice?*

2. *What is the probability John rolls the dice no more than 10 times?*

3. *What is the probability John requires between 30 and 40 attempts before hitting the sum of 12?*

Problem 7 *In 2017, 71% of all full-time KSU undergraduates received some type of need-based financial aid (https://www.usnews.com/best-colleges/kennesaw-state-university-1577). What is the average number of undergraduates that must be randomly selected to find someone without need-based financial aid?*

2 The Poisson Distribution

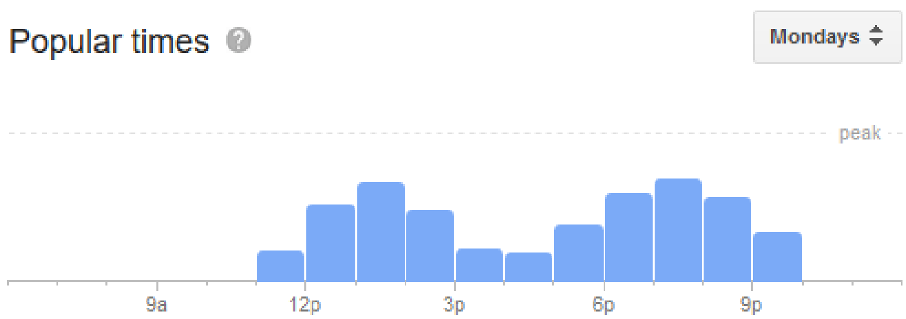
The Poisson distribution counts the number of occurrences of a particular event in a fixed unit of measurement.

Definition 8 *An experiment has a* ***Poisson Probability Distribution*** *if and only if*

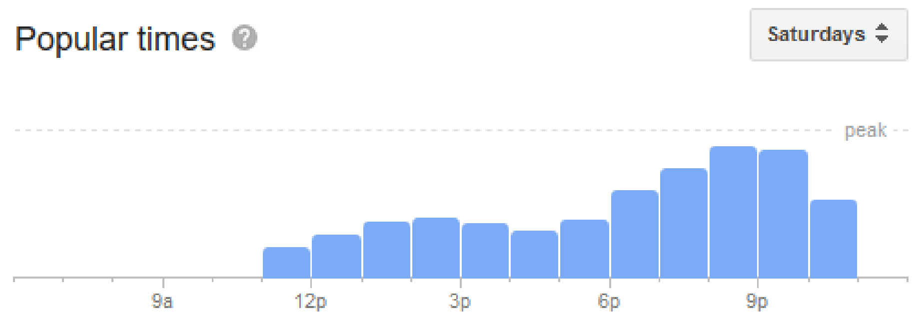
1. *the probability that a single event occurs in any fixed unit of measurement is the same for all intervals;*

2. *the mean number of events λ that occur in any interval is independent of the number that occur in any other interval.*

Example 9 *Customer dining at Rusan's on Barrett Pkwy on Mondays does not follow a Poisson distribution since the average number of customers is not fixed throughout all time periods of the day.*



Example 10 *Customer dining at Rusan's on Barrett Pkwy on from 8 PM to 9 PM during the week does not follow a Poisson distribution since the average number of customers is not fixed throughout all days.*



Example 11 *Customer dining at Rusan's on Barrett Pkwy on from 1 PM to 2 PM on Monday does follow a Poisson distribution since the average number of customers is fixed throughout all Mondays.*

Example 12 *Suppose the average number of underfilled 12 ounce cans of cola in a 12 pack is* $\lambda = .01$ *and cans are independently filled. This process follows a Poisson distribution.*

Theorem 13 *If an experiment has a Poisson probability distribution where* λ *is the average number of occurrences in any fixed unit of measurement then*

1. *the probability of* k *occurrences in a fixed period of measurement is* $p(k) = \frac{e^{-\lambda}\lambda^k}{k!}$*;*
2. *the mean or expected number of occurrences in a fixed period of measurement is* $\mu = \lambda$*;*
3. *and the standard deviation is* $\sigma = \sqrt{\lambda}$.

Remark 14 *On the TI-83/84 series of calculators,* ***poissonpdf***(λ, k) *computes the probability of* k *occurrences in any fixed unit of measurement. The command* ***poissontcdf***(λ, k) *computes the probability of* 0 *or* 1 *or* 2 *or...or* k *occurrences in any fixed unit of measurement.*

Remark 15 *In R Studio, dpois(*k*,*λ*) computes the probability of* k *occurrences in any fixed unit of measurement. The command* ***ppois***(k, λ) *computes the probability of* 0 *or* 1 *or* 2 *or...or* k *occurrences in any fixed unit of measurement.*

Example 16 *Suppose the average number of customers from 1 PM to 2 PM at Rusan's on Monday is* $\lambda = 25$.

1. *What is the standard deviation for the average number of customers from 1 PM to 2 PM at Rusan's on Monday?* $\sigma = \sqrt{25} = 5$.
2. *What is the probability that exactly 20 customers dine from 1 PM to 2 PM at Rusan's next Monday?*

$p(20) = \frac{e^{-25}25^{20}}{20!} = 5.1917 \times 10^{-2}$

```
> dpois(20,25)
[1] 0.05191747
>
```

3. *What is the probability that no more than 4 customers dine from 1 PM to 2 PM at Rusan's next Monday?*
   $p(0) + p(1) + p(2) + p(3) + p(4) = \frac{e^{-25}25^{0}}{0!} + \frac{e^{-25}25^{1}}{1!} + \frac{e^{-25}25^{2}}{2!} + \frac{e^{-25}25^{3}}{3!} + \frac{e^{-25}25^{4}}{4!} = 2.6691 \times 10^{-7}$

```
> ppois(4,25)
[1] 2.669083e-07
>
```

4. *What is the probability that between 23 and 27 customers dine from 1 PM to 2 PM at Rusan's next Monday?*
   $p(23) + p(24) + p(25) + p(26) + p(27) = \frac{e^{-25}25^{23}}{23!} + \frac{e^{-25}25^{24}}{24!} + \frac{e^{-25}25^{25}}{25!} + \frac{e^{-25}25^{26}}{26!} + \frac{e^{-25}25^{27}}{27!} = 0.38265.$

```
> ppois(27,25)-ppois(22,25)
[1] 0.3826527
>
```

Problem 17 *Suppose the average number of sips of water Dr. DeMaio consumes in a 50 minute class period is 14.*

1. *Find the mean and standard deviation for the number of sips of water Dr. DeMaio consumes in a 50 minute class period.*

2. *Find the probability that Dr. DeMaio takes less than 10 sips of water in a 50 minute class period.*

3. *Would it be unusual for Dr. DeMaio to take more than 20 sips of water in a 50 minute class period?*

4. *What percentage of the time is Dr. DeMaio's water consumption more than two standard deviations below average?*

3 Exercises

1. Kokoska Section 5.5: 5.112-5.115, 5.118-5.122, 5.124, 5.126, 5.129