

## Section 8.3: Small Sample Confidence Intervals for Population Means

If we use a sample to make an inference about a population mean then we must not know what the population mean is. If we don't know the population mean then it seems quite unlikely that we know the population standard deviation. With large samples we can use the sample standard deviation. While  $s$  may not as close to  $\sigma$  as we might like since we have a large sample, it will be OK. That is not the case with small samples. We continue to use the sample standard deviation as an estimate for the population standard deviation. When using this estimation, we are no longer guaranteed a normal distribution. Thus we end up using the **Student's  $t$  Distribution** in place of the Normal Distribution. The final form for our confidence interval for the mean with an unknown  $\sigma$  or a small sample is

$$\bar{x} \pm \text{margin of error} = \bar{x} \pm t_{\frac{\alpha}{2}} \times \frac{s}{\sqrt{n}}$$

where  $t_{\frac{\alpha}{2}}$  is a function of the level of confidence and the number of degrees of freedom =  $n - 1$ .

**Remark 1** *A confidence interval for the mean with unknown population standard deviation is computed on the TI-83/84 using the **TInterval** command from statistical tests.*

**Remark 2 *WARNING!*** *Don't use a small sample confidence interval on a data set that contains an outlier.*

**Example 1** *A sample of size  $n = 19$  yields a sample mean of 76.3 and sample standard deviation of 8.1. Construct a 95% confidence interval for the mean. Since the population standard deviation is unknown we use the student  $t$  distribution. On the calculator select **STATS** as your option and then **TInterval(76.3, 8.1, 19, .95) = (72.396, 80.204)**.*

**Exercise 1** *A sample of size  $n = 39$  yields a sample mean of 76.3 and sample standard deviation of 8.1. Construct a 95% confidence interval for the mean. Compare your result to the prior example.*

**Remark 3** *What are the  $t_{\frac{\alpha}{2}}$ -values? They are similar to  $z_{\frac{\alpha}{2}}$  values but depend on the value of  $n$ . Note that  $t_{\frac{\alpha}{2}} \geq z_{\frac{\alpha}{2}}$ . For small  $n$ ,  $t_{\frac{\alpha}{2}}$  is much larger than  $z_{\frac{\alpha}{2}}$ . As  $n$  approaches 30 from below,  $t_{\frac{\alpha}{2}}$  approaches  $z_{\frac{\alpha}{2}}$ .*

**Definition 1** *The degrees of freedom for a small sample  $t$ -test is  $n - 1$ .*

**Exercise 2** What is  $t_{\frac{\alpha}{2}}$  for a sample of size 15 at 95% confidence (using a table)? Using Table A3 in the book we first compute the degrees of freedom to be  $15 - 1 = 14$ . Table A3 is defined by the amount of error in the right tail. So, for 95% confidence there is 2.5% error in the right tail. Thus,  $t_{\frac{\alpha}{2}} = 2.145$ . In R Studio,

```
> n<-15
> alpha = 0.05
> degrees_of_freedom = n - 1
> t_score = qt(p=alpha/2, df=degrees_of_freedom,lower.tail=F)
> print(t_score)
[1] 2.144787
> |
```

**Exercise 3** What is  $t_{\frac{\alpha}{2}}$  for a sample of size 15 at 95% confidence (using a TI-84)? Here we use the **invT** command with inputs for area below the  $t_{\frac{\alpha}{2}}$  and the number of degrees of freedom. Thus, **invT**(.975, 14) = .2145. Note that **invT**(.025, 14) = -.2145.

**Exercise 4** A random sample of 57 KSU students yields a sample average driving distance from home to campus of 7.8 miles with a sample standard deviation of 4.5 miles. Construct a 99% confidence interval for the average number of miles KSU students drive to campus.

**Example 2** In the first 6 games of the 2017 season, Atlanta Falcons quarterback, Matt Ryan, has thrown for total game yards of 321, 252, 294, 242, 248 and 233. Construct a 99% confidence interval for the average number of yards per game Ryan will throw for in 2017.

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Date	Week	Tm	Opp	Result	GS	Passing										Rushing				
						Cmp	Att	Cmp%	Yds	TD	Int	Rate	Sk	Yds	Y/A	AV/A	Rush	Yds	Y/A	TD
<a href="#">2017-09-10</a>	1	ATL	@ CHI	<a href="#">W 23-17</a>	*	21	30	70.00	321	1	0	116.1	2	13	10.70	11.37	3	11	3.67	0
<a href="#">2017-09-17</a>	2	ATL	@ GNB	<a href="#">W 34-23</a>	*	19	28	67.86	252	1	0	108.0	3	29	9.00	9.71	1	8	8.00	0
<a href="#">2017-09-24</a>	3	ATL	@ DET	<a href="#">W 30-26</a>	*	24	35	68.57	294	2	3	77.6	2	17	8.40	5.69	1	-1	-1.00	0
<a href="#">2017-10-01</a>	4	ATL	@ BUF	<a href="#">L 17-23</a>	*	24	42	57.14	242	1	2	61.8	1	0	5.76	4.10	1	8	8.00	0
<a href="#">2017-10-15</a>	6	ATL	@ HIA	<a href="#">L 17-20</a>	*	24	35	68.57	248	1	1	86.4	2	9	7.09	6.37	0	0	0	0
<a href="#">2017-10-22</a>	7	ATL	@ NYE	<a href="#">L 7-23</a>	*	23	33	69.70	233	1	0	99.7	1	10	7.06	7.67	3	37	12.33	0

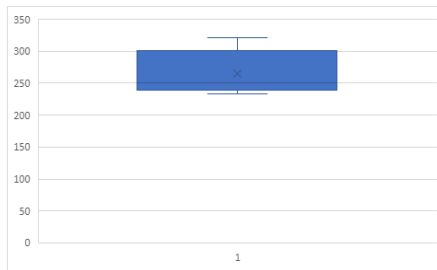
**Upcoming Games - Bye in Week 5**

<a href="#">2017-10-29</a>	8	ATL	@ NYJ	<a href="#">Preview</a>	<a href="#">Stats for 2 games vs. Jets</a>	• Avg. 26/40, 236 yds, 1.5 TD, 0.0 int (Last in 2013)
<a href="#">2017-11-05</a>	9	ATL	@ CAR	<a href="#">Preview</a>	<a href="#">Stats for 18 games vs. Panthers</a>	• Avg. 24/36, 270 yds, 1.6 TD, 0.8 int (Last in 2016)
<a href="#">2017-11-12</a>	10	ATL	@ DAL	<a href="#">Preview</a>	<a href="#">Stats for 3 games vs. Cowboys</a>	• Avg. 22/35, 275 yds, 1.3 TD, 0.7 int (Last in 2015)

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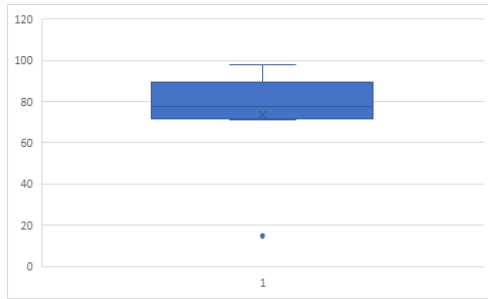
Note that when plotting this data, no outliers appear. Thus, we can use a small sample confidence interval.



On the calculator enter the sample into list  $L_1$  and then select DATA as your option in TInterval. The resulting confidence interval is (208.03, 321.97). Here's the R code.

```
> Matt_Ryan=c(321,252,294,242,249,233)
> mean_value<-mean(Matt_Ryan)
> print(mean_value)
[1] 265.1667
> n<-length(Matt_Ryan)
> print(n)
[1] 6
> standard_deviation<-sd(Matt_Ryan)
> print(standard_deviation)
[1] 34.51038
> # Find the standard error
> standard_error <- standard_deviation / sqrt(n)
> alpha = 0.01
> degrees_of_freedom = n - 1
> t_score = qt(p=alpha/2, df=degrees_of_freedom,lower.tail=F)
> margin_error <- t_score * standard_error
>
> # Calculating lower bound and upper bound
> lower_bound <- mean_value - margin_error
> upper_bound <- mean_value + margin_error
>
> # Print the confidence interval
> print(c(lower_bound,upper_bound))
[1] 208.3586 321.9747
> |
```

**Exercise 5** Dr. Jones grades eight papers from his introduction to archeology course. The scores are 91, 73, 78, 15, 84, 77, 71 and 98. Construct a 90% confidence interval for the mean population grade for this test. Note the occurrence of an outlier in this small data set. We should not construct a confidence interval from this data.



## 1 Exercises

1. Kokoska 3rd edition: 8.72, 8.73, 8.75, 8.77