Section 9.6: Hypothesis Testing for Proportions

The average is an important population parameter but by no means (pun fully intended) is it the only population parameter of note. Population proportions (or percentages) are also quite valuable. Everything we did in hypothesis testing for a population mean works the same way for population proportions with only a small change when computing the test statistic. The critical values remain the same and depend on the level of confidence and the number of tails in the test.

level of confidence	α	1-tailed test	2-tailed test
90%	0.10	1.28	1.645
95%	0.05	1.645	1.96
98%	0.02	2.05	2.33
99%	0.01	2.33	2.575

Example 1 *The KSU Dodgeball exploratory committee claims that 30% of all KSU students will attend a KSU Dodgeball game. The KSU Sentinel believes the Dodgeball exploratory committee has over-estimated their claim. Formulate the null and alternative hypothesis.*

$$H_0 : \mu_p = 0.3$$

$$H_A : \mu_p < 0.3$$

Example 2 *At 95% confidence what is the critical value?*

This is a one-tailed test. So at 95% significance, the critical value $z = 1.645$. Furthermore since we are looking at the left tail the critical value is actually $z = -1.645$.

The only difference when running a test of hypothesis for proportions rather than means is in the test statistic. The numerator will contain the difference of the sample proportion and the claimed population proportion. The denominator will contain the standard error for sample proportions. Where p is the sample percentage, the test statistic is

$$z = \frac{p - \mu_p}{\sqrt{\frac{\mu_p(1 - \mu_p)}{n}}}.$$

Example 3 *The KSU Sentinel collects a sample of 150 students where 35 students state they will attend a KSU Dodgeball game. Has the Sentinel collected statistically significant data in order to reject the exploratory committees claim?*

The sample proportion is $p = \frac{35}{150} = 0.233\,33$. The test statistic is $z = \dfrac{0.233 - 0.3}{\sqrt{\dfrac{0.3(1-0.3)}{150}}} = -1.790\,7$. The test statistic falls into the rejection region and we can reject the null hypothesis in favor of the alternative hypothesis. So, yes, the Sentinel has statistically significant evidence the exploratory committee has over-estimated their claim about the percentage of KSU students that will attend a Dodgeball game.

Example 4 *During a world series game, Mighty Casey comes up to bat. The announcers highlight his .310 batting average for the regular season but then go on to say that those numbers don't apply to the playoffs. At the end of the World Series, Mighty Casey has batted 11 for 29. Are the announcers correct or is this just an example of chance variation. Conduct a hypothesis test at 99% significance.*

$$H_0 : \mu_p = .310$$

$$H_A : \mu_p \neq .310$$

Here we use a two-tailed test since the announcers indicate that batting averages are different in the playoffs without indicating if the averages are better or worse. So, the critical value is ± 2.575. Mighty Casey's batting average (which is really a proportion) is $p = \frac{11}{29} = 0.379\,31$ The test statistic is $z = \dfrac{0.379 - 0.310}{\sqrt{\dfrac{0.310(1-0.310)}{29}}} = 0.803\,42$. The test statistic does not fall into the rejection region. We interpret this to mean that Mighty Casey is the same batter during the regular season as he is during the World Series.

Exercise 1 *A company claims that 20% of its candies are green. Eve doesn't believe that machinery can dispense proportions of candies that precisely. Eve purchases a large bag of 523 candies and finds that 99 of them are green. At 95% confidence do you agree with Eve or believe her result is just chance variation in the bag she bought?*

Exercise 2 *According to US News and World Report (http://colleges.usnews.rankingsandreviews.com/best-colleges/kennesaw-state-university-1577), "71 percent of full-time undergraduates receive some kind of need-based financial aid and the average need-based scholarship or grant award is $4,439." Bill doesn't think he knows that many students with financial aid and thinks the administrators at the University are inflating the numbers to make themselves look good. A random sample of 157 KSU students is collected and finds 123 students with financial aid of some sort. At 99% confidence do you believe Bill?*

Exercise 3 *Despite the evidence provided, Bill continues to claim that he doesn't know that many students who receive financial aid. What might account for Bill's viewpoint?*

1 Exercises

1. Kokoska Section 9.6: 9.172, 9.173, 9.179-9.185