

## Section 4.2: The Addition Rule and Rule of Complements

### 1 The Addition Rule

**Theorem 1** *If  $A$  and  $B$  are disjoint (mutually exclusive) events then  $P(A \text{ or } B) = P(A) + P(B)$ .*

**Example 1** *Pick a single card from a deck. What is the probability that you select an Ace or an 8?*

*These are disjoint events. There are four of each rank in a deck of cards. Thus, the probability that you select an Ace or an 8 is  $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = 0.15385$ .*

**Exercise 1** *Pick a single card from a deck. What is the probability that you select a club or a diamond?*

**Exercise 2** *A study at a local bar found people of various ages playing games.*

	21-29	30-39	40-49	50 and older	Total
Darts	4	12	15	6	37
Pool	8	17	16	11	52
Karaoke	17	5	0	1	23
Total	29	34	31	18	112

Find the probability that a randomly selected person...

1. is playing pool;
2. is 30-39;
3. is playing pool or singing karaoke;
4. is 21-29 or 40-49.

**Exercise 3** Recall that in the game of *Clue*, there are six suspects, six possible weapons and nine locations. The murder of Mr. Boddy was committed by one suspect, with one weapon in one location. What is the probability that Prof. Plum or Col. Mustard committed the murder?

**Exercise 4** A pair of fair dice is rolled. What is the probability that the sum of the dice is 4 or 5?

**Exercise 5** A pair of fair dice is rolled. What is the probability that the sum of the dice is at least 10?

Of course there are times when we want to determine  $P(A \text{ or } B)$  when  $A$  and  $B$  are not disjoint. In such a case we must employ the general additions rule.

## 2 The General Addition Rule

For any events  $A$  and  $B$ ,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ .

**Example 2** A card is selected at random. What is the probability it is a club or a 3?  $P(\text{club or } 3) = P(\text{club}) + P(3) - P(3 \text{ of clubs}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{4}{13} = 0.30769$ .

**Exercise 6** A card is selected at random. What is the probability it is a face card or a heart?

**Exercise 7** Roll a pair of dice. Compute the probability that

1. the sum is 7 or 8;
2. the sum is 5 or a 2 appears on at least one die.

**Exercise 8** *A study at a local bar found people of various ages playing games.*

	<b>21-29</b>	<b>30-39</b>	<b>40-49</b>	<b>50 and older</b>	<b>Total</b>
<b>Darts</b>	4	12	15	6	37
<b>Pool</b>	8	17	16	11	52
<b>Karaoke</b>	17	5	0	1	23
<b>Total</b>	29	34	31	18	112

Find the probability that a randomly selected person...

1. is playing pool or darts;
2. is 30-39 or throwing darts;
3. is playing pool or 50 and older;
4. is 21-29 or singing Karaoke.

**Definition 1** *Let  $A$  be any event. The complement of  $A$ ,  $\bar{A}$  (also  $A^c$  or  $A'$ ) is the event that  $A$  does not occur.*

**Exercise 9** *For each of the following events, describe its complement.*

1. *When flipping a coin, a head is observed.*
2. *When flipping a coin twice, exactly one head is observed.*
3. *When flipping a coin twice, at least one head is observed.*

4. When flipping a coin ten times, at least one head is observed.

**Theorem 2** For any event  $A$ ,  $P(\bar{A}) = 1 - P(A)$ .

**Exercise 10** In the game of Clue, what is the probability that Prof. Plum is innocent?

**Exercise 11** A pair of fair dice is rolled. What is the probability that the sum of the dice is less than or equal to 10?

### 3 Exercises

1. Navidi/Monk Section 4.2: 5-8, 13-37, 40, 41