

Section 6.3: The Central Limit Theorem for Proportions

1 The Central Limit Theorem and the Sampling Distribution of the Proportion

The mean is not the only statistic whose sampling distribution is normal. Many, many useful statistics have their own versions of the Central Limit Theorem.

Theorem 1 *The Central Limit Theorem (CLT for proportions)* The proportion of a random sample has a sampling distribution whose shape can be approximated by a normal model if $np \geq 10$ and $n(1-p) \geq 10$. The larger the sample, the better the approximation will be. The center of the sampling distribution, μ_p , is the population proportion p . The spread of the sampling distribution, σ_p , is $\sqrt{\frac{p(1-p)}{n}}$. The value σ_p is called the **standard error**.

Example 1 As of 6/20/2012, Ben Roethlisberger has completed 63.1% of his passes. If Ben throws 40 passes in his next game, what is the likelihood that he completes at least 60% of his passes? Note that $p = .631$, so $\sigma_p = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.631(1-.631)}{40}} = 0.076$. Using $\text{normalcdf}(.6, \infty, .631, .076)$ we get 0.658.

Example 2 If Ben throws 35 passes in his next game, what is the likelihood that he completes no more than 15 of his passes? Note that $p = .631$, but with a different number of passes $\sigma_p = \sqrt{\frac{.631(1-.631)}{35}} = 0.082$. Also, we need to convert 15 completions into a $\frac{15}{35} = 0.42857$ completion rate. Now using $\text{normalcdf}(-\infty, .428, .631, .082)$ we get 0.0067.

Problem 1 A fair coin is flipped 100 times and the percentage of heads is recorded. We should expect to see 50% = .5 of the flips be heads. Of course, where chance is involved the actual percentage of observed heads may not be exactly as expected.

1. Find σ_p .
2. A fair coin is flipped 100 times and the percentage of heads is recorded. What is the probability that at most 45% of the flips are heads?

Problem 2 A fair die is rolled 72 times and the percentage of 6s is recorded. What is the probability that at most 10% of the rolls are 6s?

Problem 3 Shaquille O'Neal has a lifetime free throw percentage rate of $p = .527$.

1. If Shaq shoots 40 free throws what is the probability that he makes at least 20 free throws?
2. If Shaq shoots 50 free throws what is the probability that he makes no more than 30 free throws?
3. If Shaq shoots 100 free throws a day, determine the number of free throws made that separates his 10% worst shooting days from the other shooting days.

Problem 4 It is estimated that 53% of all movie-watching, baseball-fans prefer the Durham Bull's Ebby Calvin (Nuke) LaLoosh over the Cleveland Indian's Ricky (Wild Thing) Vaughn. If 150 fans vote on which player is better, what is the probability that Vaughn receives at least 50% of the vote? (<https://backdoorslider.com/2015/05/08/nuke-vs-wild-thing/>)

Problem 5 A candy company claims that its candy flavor mix contains 13% dark chocolate candies. Suppose that the candies are packaged at random in large bags containing about 200 candies. What is the probability that a bag will contain more than 20% dark chocolate candies?

2 Exercises

1. Navidi/Monk: Section 6.4: 5, 6, 9, 11, 13, 15, 17