Section 5.3: Small Sample Confidence Intervals for Population Means

If we use a sample to make an inference about a population mean then we must not know what the population mean is. If we don't know the population mean then it seems quite unlikely that we know the population standard deviation. With large samples we can use the sample standard deviation. While s may not as close to α as we might like since we have a large sample, it will be OK. That is not the case with small samples. We continue to use the sample standard deviation as an estimate for the population standard deviation. When using this estimation, we are no longer guaranteed a normal distribution. Thus we end up using the **Student's t Distribution** in place of the Normal Distribution. The final form for our confidence interval for the mean with an unknown σ or a small sample is

$$\bar{x} \pm \text{ margin of error } = \bar{x} \pm t_{\frac{\alpha}{2}} \times \frac{s}{\sqrt{n}}$$

where $t_{\frac{\alpha}{2}}$ is a function of the level of confidence and the number of degrees of freedom $= n - 1$.

Remark 1 *A confidence interval for the mean with unknown population standard deviation is computed on the TI-83/84 using the **TInterval** command from statistical tests.*

Remark 2 ***WARNING!*** *Don't use a small sample confidence interval on a data set that contains an outlier.*

Example 1 *A sample of size $n = 19$ yields a sample mean of 76.3 and sample standard deviation of 8.1. Construct a 95% confidence interval for the mean.*
Since the population standard deviation is unknown we use the student t distribution. On the calculator select STATS as your option and then
$\mathbf{TInterval(76.3, 8.1, 19, .95) = (72.396, 80.204)}$.

Exercise 1 *A sample of size $n = 39$ yields a sample mean of 76.3 and sample standard deviation of 8.1. Construct a 95% confidence interval for the mean. Compare your result to the prior example.*

Remark 3 *What are the $t_{\frac{\alpha}{2}}$-values? They are similar to $z_{\frac{\alpha}{2}}$ values but depend on the value of n. Note that $t_{\frac{\alpha}{2}} \geq z_{\frac{\alpha}{2}}$. For small n, $t_{\frac{\alpha}{2}}$ is much larger than $z_{\frac{\alpha}{2}}$. As n approaches 30 from below, $t_{\frac{\alpha}{2}}$ approaches $z_{\frac{\alpha}{2}}$.*

Definition 1 *The degrees of freedom for a small sample t-test is $n - 1$.*

Exercise 2 *What is $t_{\frac{\alpha}{2}}$ for a sample of size 15 at 95% confidence (using a table)? Using Table A3 in the book we first compute the degrees of freedom to be* $15 - 1 = 14$*. Table A3 is defined by the amount of error in the right tail. So, for 95% confidence there is 2.5% error in the right tail. Thus,* $t_{\frac{\alpha}{2}} = 2.145$*.*

Exercise 3 *What is $t_{\frac{\alpha}{2}}$ for a sample of size 15 at 95% confidence (using a TI-84)? Here we use the* ***invT*** *command with inputs for area below the $t_{\frac{\alpha}{2}}$ and the number of degrees of freedom. Thus,* $\mathbf{invT}(.975, 14) = .2145$*. Note that* $\mathbf{invT}(.025, 14) = -.2145$*.*

Exercise 4 *What is $t_{\frac{\alpha}{2}}$ for a sample of size 15 at 95% confidence for a lower confidence bound (using a TI-84)? Here, $invT(.05, 14) = -1.76$. Had we used the table in the book, we would need to manually prepend the negative sign.*

Exercise 5 *A random sample of 57 KSU students yields a sample average driving distance from home to campus of 7.8 miles with a sample standard deviation of 4.5 miles. Construct a 99% confidence interval for the average number of miles KSU students drive to campus.*

Example 2 *In the first 6 games of the 2017 season, Atlanta Falcons quarterback, Matt Ryan, has thrown for total game yards of 321, 252, 294, 242, 248 and 233. Construct a 99% confidence interval for the average number of yards per game Ryan will throw for in 2017.*

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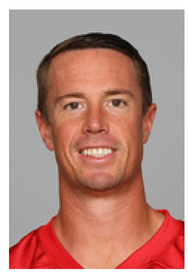
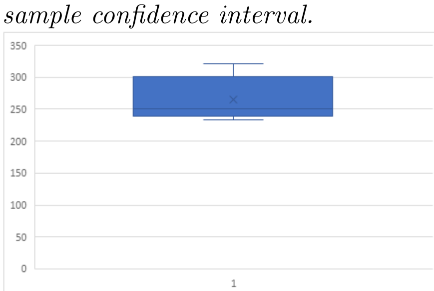
							Passing											Rushing			
Date	Week	Tm		Opp	Result	GS	Cmp	Att	Cmp%	Yds	TD	Int	Rate	Sk	Yds	Y/A	AY/A	Rush	Yds	Y/A	TD
2017-09-10	1	ATL	@	CHI	W 23-17	*	21	30	70.00	321	1	0	116.1	2	13	10.70	11.37	3	11	3.67	0
2017-09-17	2	ATL		GNB	W 34-23	*	19	28	67.86	252	1	0	108.0	3	29	9.00	9.71	1	8	8.00	0
2017-09-24	3	ATL	@	DET	W 30-26	*	24	35	68.57	294	2	3	77.6	2	17	8.40	5.69	1	-1	-1.00	0
2017-10-01	4	ATL		BUF	L 17-23	*	24	42	57.14	242	1	2	61.8	1	0	5.76	4.10	1	8	8.00	0
2017-10-15	6	ATL		MIA	L 17-20	*	24	35	68.57	248	1	1	86.4	2	9	7.09	6.37	0	0		0
2017-10-22	7	ATL	@	NWE	L 7-23	*	23	33	69.70	233	1	0	99.7	1	10	7.06	7.67	3	37	12.33	0
Upcoming Games · Bye in Week 5																					
2017-10-29	8	ATL	@	NYJ	Preview · Stats for 2 games vs. Jets · Avg. 26/40, 236 yds, 1.5 TD, 0.0 int (Last in 2013)																
2017-11-05	9	ATL	@	CAR	Preview · Stats for 18 games vs. Panthers · Avg. 24/36, 270 yds, 1.6 TD, 0.8 int (Last in 2016)																
2017-11-12	10	ATL		DAL	Preview · Stats for 3 games vs. Cowboys · Avg. 22/35, 275 yds, 1.3 TD, 0.7 int (Last in 2015)																
Falcons 2017 Stats and Schedule · Matt Ryan Splits · Matt Ryan Gamelogs · TD Passes · Game-Winning Drives · Penalties																					

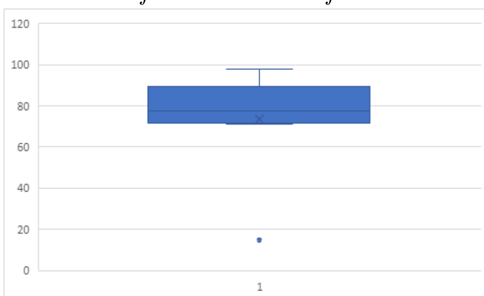


Note that when plotting this data, no outliers appear. Thus, we can use a small sample confidence interval.



On the calculator enter the sample into list L_1 and then select DATA as your option in TInterval. The resulting confidence interval is $(208.03, 321.97)$*.*

Exercise 6 *Dr. Jones grades eight papers from his introduction to archeology course. The scores are* $91, 73, 78, 15, 84, 77, 71$ *and* 98*. Construct a 90% confidence interval for the mean population grade for this test.*
Note the occurrence of an outlier in this small data set. We should not construct a confidence interval from this data.



1 Exercises

1. Navidi: 1-4, 7, 9, 12, 13