1 Computations

1. $10! =$
$\frac{500!}{502!} =$
$\binom{18}{4} =$
$\frac{\binom{2n}{2}}{\binom{2n}{3}} =$
2. Without calculating a final answer, show $16! = 14!5!2!$.

3. $\sum\limits_{i=1}^{7} 2i - 4 =$
$\sum\limits_{i=24}^{79} i^2 =$
$\prod\limits_{i=-533}^{278} \left(i^3 - 1\right)$
4. $\sum\limits_{i=1}^{n} 2^i \binom{n}{i} =$

5. List the members of the set $S = \left\{x | x \in Z^+, 50 \leq x^3 \leq 150\right\}$.

Construct $P(A)$ for $A = \{*, a, 3\}$.

Compute $|P(A)|$ for $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31\}$.

6. Give an example of sets A and B such that B is a proper subset of A and $|A| = |B|$.

7. Give an example of sets A, B and C such that $|A| = |B| = |C| = \aleph_0$ where $|A - B| = \aleph_0$ but $|A - C| = \emptyset$.

8. Use the binomial theorem to expand $(3x-2)^4$ into polynomial form. You must show all details.

9. Find the coefficient of x^{10} in the expansion of
(a) $(2x - 4)^{15}$;
(b) $\left(4x^3 - 3\right)^{12}$;
(c) $\left(8x^5 - 3\right)^5$.

10. Let $g_0 = 1$. Let $g_n = 2^{g_{n-1}}$ for $n \geq 1$. Compute g_1, g_2, g_3 and g_4.

Give a recursive definition of the set of positive integer powers of 5.

11. How many positive integers not exceeding 1000 are divisible by 6, 9 or 15?

12. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 4, 5, 8, 9\}$ and $B = \{2, 4, 5, 6, 9, 10\}$. List the members of the following sets.
i. $A \cup B =$
ii. $A \cap B =$
iii. $\overline{\bar{A} \cup B}$
iv. $A - B =$
v. $A \oplus B =$

2 Problems

1. (10 points) Shot in the back while playing poker, Wild Bill Hickok's final hand was a pair of aces and a pair of eights, now known as the dead man's hand. How many different dead man's hands exist?
2. (10 points) Using a standard deck of 52 cards (with no jokers) determine the probability of a full house.
3. (15 points) Using a standard deck of 52 cards (with no jokers) where each of the four deuces is considered a wild card, which hand should win: a five of a kind or a royal flush?
4. Prove $|Q^+| = \aleph_0$.
5. Use mathematical induction to prove $\sum\limits_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ for all $n \in Z^+$.
6. (10 points) Use induction to prove 3 divides $n^3 + 2n - 15$ for all $n \in Z^+$.
7. Use induction to prove $\sum\limits_{i=1}^{n} f_i^2 = f_n f_{n+1}$ for the Fibonacci sequence and $n \in Z^+$.

8. For sets A, B and C, let $A \oplus C = B \oplus C$. Use contradiction to prove $A = B$.

9. Use a combinatorial proof to show $\binom{4n}{2} = 4\binom{n}{2} + 6n^2$ for $n \in Z^+$.

10. A woman has 11 close friends. Find the number of ways she can invite 5 of them to dinner where:
i. There are no restrictions.
ii. Two of the friends are newlyweds and will not attend separately.
iii. Two of the friends are divorced from each other and will not attend together.

11. Two married couples, two single men and one single woman sit in a row of seven consecutive seats. How many ways can they be seated
i. with no restrictions?
ii. alternating genders?
iii. such that the women are all consecutive?
iv. such that spouses sit next to one another.

12. A social club consists of 10 pairs of twins, some identical, some fraternal. Of these 20 members, 13 are women and 7 are men.
i. How many ways can this club select a social coordinator and treasurer if these roles must be served by different club members?
ii. How many ways can this club select a social coordinator and treasurer if these roles must be served by club members of opposite gender?
iii. How many ways can this club form a six person committee to coordinate the club trip to the annual national meeting of twins? Note that each person on this committee is of equal rank and power.
iv. How many ways can this club form a six person committee that consists of three twin pairs to coordinate the club trip to the annual national meeting of twins? Note that each person on this committee is of equal rank and power.
v. How many ways can this club form a six person committee that consists of six club members but no twin pairs to coordinate the club trip to the annual national meeting of twins? Note that each person on this committee is of equal rank and power.

13. At a particular university, a student's password consists of five lowercase letters such that no letters are repeated, the first letter is a letter from the student's first name and the last letter is any vowel (no y's allowed), not necessarily from the student's first name. How many different passwords can Mark create?

14. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels and plain bagels.

How many ways are there to choose
i. 6 bagels;
ii. 2 dozen bagels;
iii. a dozen bagels with at least one of each kind?

15. Susan buys an economy pack of fifty pens. The pens are identical except for color. There are ten of each of five different colors.
i. How many different ways can Susan select four pens of different colors to take to work?
ii. How many different ways can Susan select ten pens to take to work?
iii. How many different ways can Susan select twelve pens to take to work with at least one of each color pen?
iv. How many different ways can Susan select six pens to take to work with at least two different colors in the mix?
v. How many different ways can Susan select twelve pens to take to work?

16. True or False: $\binom{n}{j+k} = \binom{n}{j} + \binom{n}{k}$ for all $n, j, k \in \mathbb{Z}^+$. If true, prove it. If false, provide a counterexample.

17. True or False: $j! + k! = (j + k)!$ for all $j, k \in \mathbb{Z}^+$. If true, prove it. If false, provide a counterexample.