Math 4322 Quiz I
DeMaio Spring 2010

Name____________________

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (5 points each) Complete the following.

   The graph K_{35} has $\binom{35}{2} = 595$ edges.

   The graph N_{72} has 0 edges.

   The graph P_{42} has 41 edges.

   The graph C_{112} has 112 edges.

   The graph W_{105} has 210 edges.

   The graph W_{105} has 106 vertices.

   The graph $K_{15,17}$ has $15 * 17 = 255$ edges.

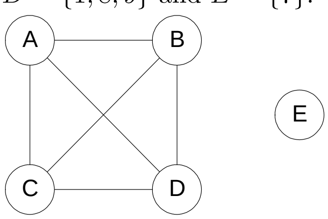
   The graph $K_{15,17}$ has $15 + 17 = 32$ vertices.

   The graph Q_6 has $2^6 = 64$ vertices.

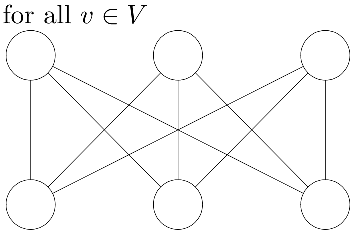
   The graph C_n is bipartite when n is even.

   The graph K_n is bipartite when $n = 1, 2$.

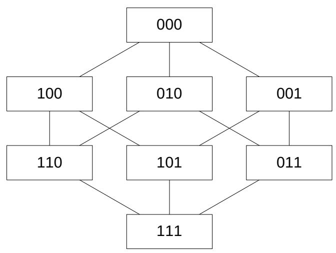
2. (10 points) Draw the intersection graph for sets $A = \{1, 4, 5, 8, 9\}$, $B = \{2, 4, 5, 6, 9, 10\}$, $C = \{1, 2, 3\}$, $D = \{1, 8, 9\}$ and $E = \{7\}$.



3. (10 points) Construct a graph $G = (V, E)$ with $n = 6$ vertices and $e = 9$ edges such that $\deg(v) \leq 3$ for all $v \in V$



4. (10 points) i. Draw Q_3. Be sure to label the vertices as bit strings.



Q_3

(5 points) ii. State the handshaking lemma.
Let $G = (V, E)$ be a graph. $\sum_{v \in V} \deg(v) = 2e$

(10 points) iii. Use the handshaking lemma to construct a formula for the number of edges in Q_n.
First, note that Q_n has 2^n vertices. Second, we must note that every vertex in Q_n is adjacent to exactly n vertices. Thus, $\sum_{v \in V} \deg(v) = n2^n$ which is also $2e$. Thus, $e = n2^{n-1}$.

5. (10 points) Suppose a graph has 150 edges, 30 vertices of degree 4, and all others of degree 3. How many vertices does the graph have?
If $e = 150$ then on the one hand, $\sum_{v \in V} \deg(v) = 2e = 300$. Let x be the number of vertice of degree 3. On the other hand $\sum_{v \in V} \deg(v) = 4 * 30 + 3x$. So, $300 = 120 + 3x$ and $x = 60$. Thus, there are $30 + 60 = 90$ vertices in the graph.