Math 3322 Quiz I
DeMaio Fall 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (10 points) Let $S = \{x | x \in \mathbb{R}^-, x$ is a solution of $\left(x^2 - 4\right)(2x + 7) = 0\}$. List the elements of S.
   The solutions of $\left(x^2 - 4\right)(2x + 7) = 0$ are $2, -2$ and $-\frac{7}{2}$. The set S consists of those solutions from the set of negative real numbers and $S = \{-2, -\frac{7}{2}\}$.

2. (10 points) Use set builder notation to define the set of all rational numbers.
   $\mathbb{Q} = \{\frac{a}{b} | a, b \in \mathbb{Z}, b \neq 0\}$

3. (10 points) Let $A = \{1, 2, 3\}$. Construct $P(A)$.
   $P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

4. (15 points) Compute the cardinality of each of the following sets.
   i. $|\emptyset| = 0$;
   ii. $B = \{1, \beta, c, \{1\}, \{\{\alpha, \beta, \gamma, \delta\}\}, \mathbb{Z}, \mathbb{Z}^+\}$;
   $|B| = 7$
   iii. $P(C)$ for $C = \{1, 2, 3, ..., 15\}$.
   $|P(C)| = 2^{|C|} = 2^{15} = 32\,768$

5. (30 points) Let $U = \{1, 2, 3, ..., 10\}$, $A = \{1, 5, 8, 9\}$, $B = \{2, 3, 7\}$ and $C = \{1, 7, 10\}$. List the members of each of the following sets.
   i. $A \cap B = \emptyset$
   ii. $A \cap C = \{1\}$
   iii. $\overline{A \cup B} =$
   $A \cup B = \{1, 2, 3, 5, 7, 8, 9\}$ so $\overline{A \cup B} = \{4, 6, 10\}$
   iv. $\overline{A} \cup \overline{B} = \{1, 2, 3, ..., 10\} = U$
   v. $(A \cup B) \cap C = \{1, 7\}$
   vi. (As defined in the homework) $A \oplus C = \{5, 7, 8, 9, 10\}$

6. (20 points) Let A and B be sets.
   i. State the definition of A is a **subset** of B.
   A is a subset of B if for every $x \in A$, $x \in B$.
   ii. What is the difference in meaning of $A \subseteq B$ versus $A \subset B$?
   The notation $A \subseteq B$ allows $A = B$ while $A \subset B$ forces A to be a proper subset of B.
   iii. Let $A \subseteq B$.
   $A \cup B = B$
   $A \cap B = A$
   $A - B = \emptyset$

7. (15 points) Prove $\sqrt{13}$ is irrational.
   Prove $\sqrt{13}$ is irrational.
   Assume $\sqrt{13}$ is rational. This allows us to write $\sqrt{13} = \frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$. Furthermore we can assume that a and b share no factors since we can always just reduce the fraction to its simplest terms. Now, if $\sqrt{13} = \frac{a}{b}$ then $13 = \frac{a^2}{b^2}$ and $a^2 = 13b^2$. So 13 divides a^2. Since 13 is prime we know that 13 must also divide a and we can write $a = 13k$ for some $k \in \mathbb{Z}$. Now we rewrite $a^2 = 13b^2$ as $(13k)^2 = 13b^2$ and see that $b^2 = \frac{(13k)^2}{13} = 13k^2$. So 13 divides b^2. Once again, since 13 is prime we know that 13 must also divide b. This, however, is a contradiction since a and b share no common factors. So, the assumption that $\sqrt{13}$ is rational is false and we've shown that $\sqrt{13}$ is irrational.