Math 3322 Quiz I
DeMaio Fall 2010

Name______________________________

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (10 points) Let $S = \{x | x \in \mathbb{Z}^{-}, x \text{ is a solution of } \left(x^2 - 4\right)(2x+7) = 0\}$. List the elements of S.
   $S = \{-2\}$

2. (10 points) Use set builder notation to define the set of all rational numbers.
   $\mathbb{Q} = \left\{ \frac{a}{b} | a, b \in \mathbb{Z}, b \neq 0 \right\}$

3. (10 points) Let $A = \{\emptyset, 1\}$. Construct $P(A)$.
   $P(A) = \{\emptyset, \{\emptyset\}, \{1\}, \{\emptyset, 1\}\}$

4. (15 points) Compute the cardinality of each of the following sets.
   i. $\emptyset$;
   $|\emptyset| = 0$
   ii. $B = \{1, 2, 3, \{\alpha, \omega, 1\}, \beta, c, \{1\}, \{\{\alpha, \beta, \gamma, \delta\}\}, \mathbb{Z}, \mathbb{Z}^{+}\}$;
   $|B| = 10$
   iii. $P(C)$ for $C = \{1, 2, 3, ..., 8\}$.
   $|P(C)| = 2^{|C|} = 2^8 = 256$

5. (35 points) Let $S = \{1, 3, \{2, 3\}, \{\emptyset, 2\}\}$. Answer the following without explanation.
   i. Is $1 \subseteq S$? No.
   ii. Is $\{2, 3\} \subseteq S$? No.
   iii. Is $\{1, 3\} \in S$? No.
   iv. Is $\emptyset \in S$? No.
   v. Is $\{\emptyset\} \in S$? No.
   vi. Is $\{3, \{2, 3\}\} \subseteq S$? Yes.
   vii. Is $(3, \{2, 3\}) \in S \times S$? Yes.

6. Let A and B be sets.
   i. (5 points) State the definition of A is a **subset** of B.
   The set A is a subset of the set B if for every $x \in A$, $x \in B$.
   ii. (5 points) What is the difference in meaning of $A \subseteq B$ versus $A \subset B$?

   If $A \subseteq B$ then A might equal B. In $A \subset B$, $A \neq B$.
   iii. (5 points) Give an example of two sets A and B such that $A \in B$ and $A \subseteq B$.
   Let $A = \{1\}$ and $B = \{1, \{1\}\}$.

7. (15 points) Prove $\sqrt{7}$ is irrational. .
   Proof by contradiction.
   Assume $\sqrt{7}$ is rational. This allows us to write $\sqrt{7} = \frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$. Furthermore we can assume that a and b share no factors since we can always just reduce the fraction to its simplest terms. Now, if $\sqrt{7} = \frac{a}{b}$ then $7 = \frac{a^2}{b^2}$ and $a^2 = 7b^2$. So 7 divides a^2. Since 7 is prime we know that 7 must also divide a and we can write $a = 7k$ for some $k \in \mathbb{Z}$. Now we rewrite $a^2 = 7b^2$ as $(7k)^2 = 7b^2$ and see that $b^2 = \frac{(7k)^2}{7} = 7k^2$. So 7 divides b^2. Once again, since 7 is prime we know that 7 must also divide b. This, however, is a contradiction since a and b share no common factors. So, the assumption that $\sqrt{7}$ is rational is false and we've shown that $\sqrt{7}$ is irrational.