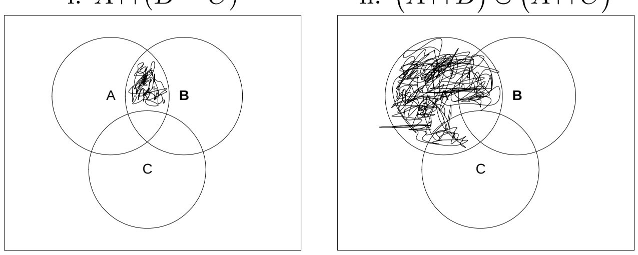
Math 3322 Quiz I
DeMaio Spring 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (10 points) List the members of the set $S = \{x | x \leq 100 \text{ and } \sqrt[3]{x} \in \mathbb{Z}^+\}$.
   $S = \{1, 8, 27, 64\}$
2. (10 points) Use set builder notation to give a description of $S = \{..., -15, -10, -5, 0, 5, 10, 15, ...\}$.
   $S = \{5x | x \in \mathbb{Z}\}$
3. (10 points) Find $P(A)$ for $A = \{1, a, \square\}$.
   $P(A) = \{\emptyset, \{1\}, \{a\}, \{\square\}, \{1, a\}, \{1, \square\}, \{a, \square\}, \{1, a, \square\}\}$
4. (15 points) What is the cardinality of each of the following sets?
   i. $|\emptyset| = 0$
   ii. $|\{a, b, \{a\}, \{a, b\}, \emptyset, \mathbb{R}, \mathbb{Z}\}| = 7$
   iii. $P(A)$ for $A = \{1, 2, 3, a, b, c, \square, \triangle, \Diamond\}$
   $|P(A)| = 2^9 = 512$
5. (10 points) Complete the definitions.
   i. Two sets A and B are **disjoint** if $A \cap B = \emptyset$.
   ii. Set A is a **subset** of set B if for every $x \in A$, $x \in B$.
6. (25 points) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 4, 5, 8, 9\}$ and $B = \{2, 4, 5, 6, 9, 10\}$. List the members of the following sets.
   i. $A \cup B = \{1, 2, 4, 5, 6, 8, 9, 10\}$
   ii. $A \cap B = \{4, 5, 9\}$
   iii. $\bar{A} \cup B = \{2, 3, 4, 5, 6, 7, 9, 10\}$
   iv. $A - B = \{1, 8\}$
   v. $A \oplus B = \{1, 2, 6, 8, 10\}$
7. (20 points)In a Venn diagram shade
   i. $A \cap (B - C)$ ii. $\left(A \cap \overline{B}\right) \cup \left(A \cap \overline{C}\right)$



8. Let A and B be sets.
   i. (5 points) Describe the strategy of the proof technique one uses to show that $A = B$.
   There are two parts to this proof. First show $A \subseteq B$. Second show $B \subseteq A$.
   ii. (10 points) Prove $A - B = A \cap \overline{B}$.
   First show $A - B \subseteq A \cap \overline{B}$. Let $x \in A - B$. This means that $x \in A$ and $x \notin B$. Thus $x \in A$ and $x \in \bar{B}$ which shows $x \in A \cap \overline{B}$.
   Second show $A \cap \overline{B} \subseteq A - B$. Let $x \in A \cap \overline{B}$. This shows $x \in A$ and $x \in \overline{B}$. Thus, $x \in A$ and $x \notin B$ which shows $x \in A - B$.
   Put both parts together and $A - B = A \cap \overline{B}$.■