Math 3322 Quiz II
DeMaio Fall 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (10 points) Compute the following.
   i. $\lfloor 1.1 \rfloor = 1$
   ii. $\lceil -1.1 \rceil = -1$
   iii. $\left\lfloor \frac{1}{2} + \left\lfloor \frac{3}{2} \right\rfloor \right\rfloor = 1$

2. (5 points) Let A and B be sets. Describe the proof strategy to show that $A = B$.
   First show $A \subseteq B$. Second show $B \subseteq A$.

3. (15 points) Let A and B be sets. Prove $A \oplus B = (A - B) \cup (B - A)$.
   First show $A \oplus B \subseteq (A - B) \cup (B - A)$. Let $x \in A \oplus B$. This implies that ($x \in A$ and $x \notin B$) or ($x \notin A$ and $x \in B$). If $x \in A$ and $x \notin B$ then $x \in A - B$ which forces $x \in (A - B) \cup (B - A)$. On the other hand if $x \notin A$ and $x \in B$ then $x \in B - A$ which also forces $x \in (A - B) \cup (B - A)$. So, $A \oplus B \subseteq (A - B) \cup (B - A)$. Now show $(A - B) \cup (B - A) \subseteq A \oplus B$. Let $x \in (A - B) \cup (B - A)$. This implies that $x \in A - B$ or $x \in B - A$. If $x \in A - B$ then $x \in A$ but $x \notin B$. However, $x \in A$ but $x \notin B$ puts $x \in A \oplus B$. Similarly, f $x \in B - A$ then $x \in B$ but $x \notin A$. However, $x \in B$ but $x \notin A$ puts $x \in A \oplus B$. So, $(A - B) \cup (B - A) \subseteq A \oplus B$. Put the two steps together and $A \oplus B = (A - B) \cup (B - A)$.

4. (10 points) Why is $f(x) = \frac{1}{x^2-2}$ a function from $\mathbb{Z}$ to $\mathbb{R}$ but not a function from $\mathbb{R}$ to $\mathbb{R}$?
   The quotient $\frac{1}{x^2-2}$ is undefined at $x = \pm\sqrt{2}$ which are real numbers but not integers. Thus, $f(x) = \frac{1}{x^2-2}$ is defined on $\mathbb{Z}$ but not on $\mathbb{R}$.

5. (10 points) i. Is floor a one-to-one function from $\mathbb{R}$ to $\mathbb{Z}$? Explain.
   No! $\lfloor 1.2 \rfloor = \lfloor 1.3 \rfloor$ but $1.2 \neq 1.3$.
   ii. Is floor an onto function from $\mathbb{R}$ to $\mathbb{Z}$? Explain.
   Yes! Take any $n \in \mathbb{Z}$. Clearly $\lfloor n \rfloor = n$ and floor is an onto function.

6. (15 points) Find the terms of the sequence $\{a_n\}$, where $a_n = 2(-3)^n + 5^n$.
   i. $a_0 = 2(-3)^0 + 5^0 = 3$
   ii. $a_2 = 2(-3)^2 + 5^2 = 43$
   iii. $a_5 = 2(-3)^5 + 5^5 = 2639$

7. (25 points) Compute the following.
   i. $\sum\limits_{i=1}^{164} i = \frac{164*165}{2} = 13\,530$
   ii. $\sum\limits_{i=1}^{164} i^2 = \frac{164*165(2*164+1)}{6} = 1483\,790$
   iii. $\sum\limits_{i=156}^{934} i = \sum\limits_{i=1}^{934} i - \sum\limits_{i=1}^{155} i = 436\,645 - 12\,090 = 424\,555$
   iv. $\prod\limits_{i=1}^{5} 2^{2i-1} = 2^1 2^3 2^5 2^7 2^9 = 2^{25} = 33\,554\,432$
   v. $\prod\limits_{i=1}^{50} \left(i^2 - 100\right) = 0$. Focus on the $i = 10$ term.

8. (10 points) Let A be the collection of all integers that are multiples of 7. Prove $|A| = \aleph_0$.
   Since we can write A as the ordered sequence $0, 7, -7, 14, -14, 21, -21, \ldots$, A can put put into a one-to-one and onto correspondence with $\mathbb{Z}^+$. Thus, $|A| = \aleph_0$.

9. (10 points) True or False? $(n + k)! = n! + k!$. If true, prove it. If false, provide a counter example.
   False! Consider $n = 2$ and $k = 3$. Note $(2 + 3)! = 5! = 120$ while $2! + 3! = 8$.