Math 3322 Quiz II
DeMaio Fall 2010

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (35 points) Compute the following.
   i. $\left\lfloor \left(\frac{1}{2}\right)^{25} \right\rfloor = 0$ ii. $\lceil -0.125 \rceil = 0$ iii. $\left\lfloor \frac{1}{2} \left\lfloor \frac{3}{2} \right\rfloor \right\rfloor = 0$
   iv. $\sum_{i=1}^{164} i = 13\,530$ v. $\sum_{i=1}^{164} i^2 = 1483\,790$ vi. $\sum_{i=156}^{934} i = 424\,555$
   vii. $\prod_{k=1}^{150} \frac{k+1}{k} = 151$

2. (10 points) Find the domain and range of the function that assigns to each bit string twice the number of zeroes in that string.
   The domain is the collection of all bit strings. The range is the set of all non-negative even integers.

3. (10 points) Find the first 10 terms of the sequence whose n^{th} term is the largest integer k such that $k! \leq n$.
   $1, 2, 2, 2, 2, 3, 3, 3, 3, 3$

4. (10 points) Let $|A| = |B| = \aleph_0$. Show that $|A \cup B| = \aleph_0$.
   Since A is countable infinite we can list every element in A in order as $a_1, a_2, a_3 ....$ Ditto for B. Now the sequence $a_1, b_1, a_2, b_2, a_3, b_3, ...$ lists every element of $A \cup B$ in order. Thus, $|A \cup B| = \aleph_0$.

5. (15 points) Use mathematical induction to prove $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ for all $n \in Z^+$.

   I. Show that $S(1)$ is true. L.H.S. $\sum_{i=1}^{1} i^3 = 1^3 = 1$. R.H.S. $\frac{1^2(1+1)^2}{4} = \frac{4}{4} = 1$. Thus, $S(1)$ is true.

   II. Assume $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ and show $\sum_{i=1}^{n+1} i^3 = \frac{(n+1)^2(n+2)^2}{4}$.
   $\sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3$ which by the inductive hypothesis is $\frac{n^2(n+1)^2}{4} + (n+1)^3 = \frac{n^2(n+1)^2}{4} + \frac{4(n+1)^3}{4} = \frac{(n+1)^2\left[n^2+4(n+1)\right]}{4} = \frac{(n+1)^2\left[n^2+4n+4\right]}{4} = \frac{(n+1)^2(n+2)^2}{4}$.

6. (10 points) Let A be the collection of all integers that are multiples of 7. Prove $|A| = \aleph_0$.
   All we need to do is list every multiple of 7 in order. The sequence $0, 7, -7, 14, -14, 21, -21, ...$ achieves that goal.

7. (10 points) True or False? $\frac{(n+k)!}{k!} = n! - k!$ for $2 < k < n$ where $n, k \in \mathbb{Z}^+$ If true, prove it. If false, provide a counter example.
   False. Let $k = 3$ and $n = 4$.
   $\frac{(3+4)!}{3!} = 840$ while $4! - 3! = 18$.

8. (10 points) Let A and B be sets. Prove $A \subseteq (A \oplus B) \oplus B$.
   Let $x \in A$. Either $x \in B$ or $x \notin B$. This leads to two cases to consider.
   1. If $x \in B$ then $x \notin (A \oplus B)$ since x would be in both A and B. However, since $x \in B$ and $x \notin (A \oplus B)$ then $x \in (A \oplus B) \oplus B$.
   2. If $x \notin B$ then $x \in (A \oplus B)$ since $x \in A$ but not in B. Furthermore since $x \in (A \oplus B)$ but not in B then $x \in (A \oplus B) \oplus B$.
   In either case, $x \in (A \oplus B) \oplus B$ and $A \subseteq (A \oplus B) \oplus B$.