Math 3322 Quiz II
DeMaio Spring 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (35 points) Compute each of the following.
   i. $7! = 5040$
   ii. $\lceil 5.39 \rceil = 6$
   iii. $\lfloor -4.55 \rfloor = -5$
   iv. $\sum_{i=5}^{9} (3i-4) = 85$
   v. $\frac{152!}{150!} = 152 * 151 = 22\,952$
   vi. $\lceil 5.6 + \lfloor \pi \rfloor \rceil = 9$
   vii. $\prod_{i=1}^{10} \left(i^3 - 125\right) = 0$

2. (10 points) State the formula for the sum of the first n integers and use it to compute $\sum_{i=17}^{57} i$.
   $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
   $\sum_{i=17}^{57} i = \sum_{i=17}^{57} i + \sum_{i=1}^{16} i - \sum_{i=1}^{16} i = \sum_{i=1}^{57} i - \sum_{i=1}^{16} i = \frac{57*58}{2} - \frac{16*17}{2} = 1653 - 136 = 1517$

3. (10 points) State the formula for the sum of the cubes of the first n integers and use it to compute $\sum_{i=1}^{7} i^3$.
   $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$
   $\sum_{i=1}^{7} i^3 = \left(\frac{7*8}{2}\right)^2 = 784$

4. (5 points) Find the domain and range of the function that assigns to each non-negative integer its last digit.
   The domain is $\mathbb{Z}^+ \cup \{0\}$ and the range is $\{0,1,2,3,4,5,6,7,8,9\}$.

5. (10 points) Give an example of a function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ that is one-to-one but not onto.
   Let $f(n) = 2n$. The function $f(n)$ is one-to-one since $2n = 2m$ implies that $n = m$. The function $f(n)$ is not onto since odd positive integers have no pre-image.

6. (5 points) For $a_n = 2(-3)^n + 5n$ compute a_1 and a_4.
   $a_1 = 2(-3)^1 + 5 * 1 = -1$
   $a_4 = 2(-3)^4 + 5 * 4 = 182$

7. (5 points) Find a simple rule that generates the sequence $15, 8, 1, -6, -13, -20, -27, ...$
   $a_n = 22 - 7n$ or $a_n = 15 - 7(n-1)$

8. (10 points) Let A be the set of all integer multiples of 3. Show $|A| = \aleph_0$.
   The function $f : \mathbb{Z} \to 3\mathbb{Z}$ via $f(n) = 3n$ is a one-to-one and onto function.

9. (10 points) Let A and B be countable sets. Prove $A \cup B$ is countable.
   If A and B are countable sets then both can be written as $A = \{a_1, a_2, a_3, ...\}$ and $B = \{b_1, b_2, b_3, ...\}$. Now $A \cup B$ can be written as $\{a_1, b_1, a_2, b_2, a_3, b_3, ...\}$.

10. (5 points) Give an example of a set that is uncountable. Among others, $\mathbb{R}$ or $(0,1)$ are the most common examples.

11. (10 points) Give an example of two sets A and B such that A is a proper subset of B yet $|A| = |B|$.
    The sets $\mathbb{Z}$ and all integer multiples of 3 from question 8 form one possible example. Note that your sets must both be infinite.