Math 3322 Quiz II
DeMaio Spring 2010

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (10 points) Give a recursive definition of the even positive integers.
   Let $E_n = E_{n-1} + 2$ for $n \geq 2$ where $E_1 = 2$.

2. (10 points) Let $r_n = (r_{n-1})^2 + 1$ where $r_1 = 1$. Compute r_2, r_3, r_4 and r_5.
   $r_2 = (r_1)^2 + 1 = (1)^2 + 1 = 2$
   $r_3 = (r_2)^2 + 1 = (2)^2 + 1 = 5$
   $r_4 = (r_3)^2 + 1 = (5)^2 + 1 = 26$
   $r_5 = (r_4)^2 + 1 = (26)^2 + 1 = 677$

3. (5 points) How many bit strings of length 8 exist? $2^8 = 256$
   (5 points) How many bit strings of length 8 exist that begin with 10 and end with 01? $2^4 = 16$
   (5 points) How many bit strings of length 8 exist that alternate 0's and 1's? 2

4. (5 points) A particular brand of shirt comes in 12 colors, has a male and female version and comes in three sizes for each sex. How many different types of this shirt are made? $12 * 2 * 3 = 72$

5. (5 points) How many different passwords exist using four lowercase letters followed by two digits (0-9)? $26^4 * 10^2 = 45\,697\,600$
   (5 points) How many different passwords exist using four distinct lowercase letters followed by two distinct digits (0-9)? $26 * 25 * 24 * 23 * 10 * 9 = 32\,292\,000$
   (5 points) How many different passwords exist using four distinct lowercase or uppercase letters followed by two distinct digits (0-9)? $52 * 51 * 50 * 49 * 10 * 9 = 584\,766\,000$

6. (5 points) In an attempt to raise productivity the CANE corporation is scheduled to publicly flog its six least productive employees. In how many different orders can these six employees be made an example of? $6! = 720$

7. (20 points) Use induction to prove $\frac{3^{2n} - 5^{2n+2}}{8} \in \mathbb{Z}$ for $n \in \mathbb{Z}^+$.
   I. Show $S(1)$ is true. So, $\frac{3^{2*1} - 5^{2*1+2}}{8} = -77$.
   II. Show that if $S(n)$ is true then $S(n+1)$ is also true. Assume $\frac{3^{2n} - 5^{2n+2}}{8} \in \mathbb{Z}$. Show $\frac{3^{2(n+1)} - 5^{2(n+1)+2}}{8} \in \mathbb{Z}$. Note that $\frac{3^{2(n+1)} - 5^{2(n+1)+2}}{8} = \frac{3^{2n+2} - 5^{2n+4}}{8} = \frac{9*3^{2n} - 25*5^{2n+2}}{8} = \frac{9*3^{2n} - 9*5^{2n+2}}{8} - \frac{16*5^{2n+2}}{8} = 9\left(\frac{3^{2n} - 5^{2n+2}}{8}\right) - 2 * 5^{2n+2} = 9 * \text{integer} + \text{integer} = \text{integer}$.

8. (10 points) State the definition of the Fibonacci sequence.
   $F_n = F_{n-1} + F_{n-2}$ where $F_0 = 0$ and $F_1 = 1$.

9. (20 points) Let B_n be the number of bit strings of length n that contain no consecutive 1's. Find and prove the correctness of a formula for B_n.
   $B_1 = 2$

$B_1 = 2$	$B_2 = 3$	$B_3 = 5$	$B_4 = 8$
0	00	000	0000
1	10	100	1000
	01	010	0100
		001	0010
		101	0001
			1001
			1010
			0101

It appears that $B_n = F_{n+2}$. In order to prove this we need to show that $B_n = B_{n-1} + B_{n-2}$. Partition all B_n bit strings into two sets: those that end in 0 and those that end in 1. There are B_{n-1} strings that end in 0 since you can add a 0 to end end of every bit string of length $n-1$. Those bits strings that end in 1, actually end in 01. There are B_{n-2} of these since you can add 01 to the end of every bit string of length $n-2$. Thus, $B_n = B_{n-1} + B_{n-2}$.