Math 3322 Quiz II
DeMaio Summer 2009

Name_____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (15 points) How many bit strings of length 8 exist? $2^8 = 256$
   How many bit strings of length 8 exist that begin with 10 and end with 01? $2^4 = 16$
   How many bit strings of length 8 exist that alternate 0's and 1's? There are 2: 10101010 and 01010101.

2. (5 points) A particular brand of shirt comes in 12 colors, has a male and female version and comes in three sizes for each sex. How many different types of this shirt are made? $12 * 2 * 3 = 72$.

3. (20 points) How many different ways can you rearrange the letter in the word
   i. game; $4! = 24$
   ii. football; $\frac{8!}{2!2!} = 10\,080$
   iii. steelers; $\frac{8!}{3!2!} = 3360$
   iv. roethlisberger? $\frac{14!}{3!3!} = 2421\,619\,200$

4. (25 points) In the game of Clue, there are six suspects (Col. Mustard, Prof. Plum, Mr. Green, Mrs. Peacock, Miss Scarlet and Mrs. White), six possible weapons (Knife, Candlestick, Revolver, Rope, Lead Pipe and Wrench) and nine locations (Hall, Lounge, Dining Room, Kitchen, Ball Room, Conservatory, Billiard Room, Library and Study). The murder of Mr. Boddy was committed by one suspect, with one weapon in one location.
   i. How many different possible ways could the murder have been committed? $6 * 6 * 9 = 324$
   ii. How many ways could Miss Scarlet have committed the murder? $1 * 6 * 9 = 54$
   iii. How many different possible ways could the murder have been committed by Col. Mustard or Mrs. Peacock? These are disjoint cases. So, $1 * 6 * 9 + 1 * 6 * 9 = 108$.
   iv. How many different possible ways could the murder have been committed by Mr. Green and using the knife? $1 * 1 * 9 = 9$
   v. How many different possible ways could the murder have been committed by Mrs. White or using the rope? These are not disjoint cases. So, $1 * 6 * 9 + 6 * 1 * 9 - 1 * 1 * 9 = 99$

5. (15 points) Let $D = \{1, 2, 3, 4\}$ and $R = \{a, b, c, d, e, f\}$.
   i. How many functions $f : D \to R$ exist? $6^4 = 1296$
   ii. How many one-to-one functions $f : D \to R$ exist? $\frac{6!}{(6-4)!} = 360$
   iii. How many onto functions $f : D \to R$ exist? 0

6. (10 points) How many positive integers not exceeding 1000 are divisible by either 4 or 6? $\left\lfloor \frac{1000}{4} \right\rfloor + \left\lfloor \frac{1000}{6} \right\rfloor - \left\lfloor \frac{1000}{12} \right\rfloor = 333$

7. (10 points) State both the Pigeonhole Principle and the generalized Pigeonhole Principle.
   Pigeonhole Principle: If $k+1$ pigeons are placed into k pigeonholes then at least one pigeonhole contains at least two pigeons.
   Genealized Pigeonhole Principle: If n pigeons are placed into k pigeonholes then at least one pigeonhole contains at least $\left\lceil \frac{n}{k} \right\rceil$ pigeons.

8. (5 points) Explicitly use the Pigeonhole Principle to show that in a class of 30 students then at least two students have last names that begin with the same letter.
   Pigeons: 30 student's last names.
   Pigeonholes: 26 letters of the alphabet.
   We place the 30 pigeons into 26 pigeonholes based on the first leter of the student's last name and at least one pigeonhole contains at least 2 pigeons.

9. (10 points) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
   i. How many balls must she select to be certain that she has at least 3 balls of the same color? Five since $\left\lceil \frac{4}{2} \right\rceil = 2$ and $\left\lceil \frac{5}{2} \right\rceil = 3$.
   ii. How many balls must she select to be certain that she has at least 3 red balls? 13