Math 4322 Quiz III
DeMaio Spring 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

Chessboard Domination

1. (10 points) Demonstrate the Spencer-Cockayne construction for dominating the 9×9 chessboard with queens.

			Q					
						Q		
				Q				
		Q						
					Q			

2. (10 points) Demonstrate the Welch upper bound construction for dominating the 11×11 chessboard with queens.

						Q				
							Q			
								Q		
	Q									
		Q								
Q										
									Q	
										Q

3. (20 points) Find each of the following:
$\gamma(K_5) = \lceil \frac{5}{3} \rceil^2 = 4$
$\gamma(Q_8) = 5$
$\gamma(B_{23}) = 23$
$\gamma(R_{5,23}) = 5$
$\gamma(K_{12,35}) = \lceil \frac{12}{3} \rceil * \lceil \frac{35}{3} \rceil = 48$

4. (10 points) Prove $\gamma(N_{4,5}) = 4$. The dominated board below shows that $\gamma(N_{4,5}) \leq 4$.

		N		
		N		
		N		
		N		

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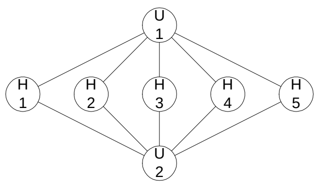
Only knights on squares $(2,3)$ and $(3,3)$ can attack 6 squares. So knights on those squares dominate 7 squares each. A knight on any other square can attack at most 4 squares and dominate at most 5 squares. Thus, the maximum number of squares three knights can dominate on the 4×5 board is

$2 * 7 + 5 = 19$. Hence, three knights cannot dominate the 20 squares of the 4×5 board. This shows $\gamma(N_{4,5}) > 3$. Put the two facts together and $\gamma(N_{4,5}) = 4$.

5. (10 points) Prove $\gamma(R_n) = n$.
   By placing a rook on each of the n squares on the first row we dominate the board and show $\gamma(R_n) \leq n$. Now assume that $n-1$ rooks are placed on the board. This forces the existence of at least one row (call it i) without a rook and at least one column (call it j) without a rook. This demonstrates that square (i, j) is not dominated. Hence, $\gamma(R_n) > n - 1$. Put these two facts together and $\gamma(R_n) = n$.

Planar Graphs

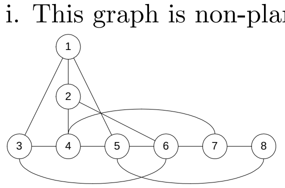
6. (10 points) Can five houses be connected to two utilities without connections crossing? If yes, prove it. If no, prove why not.
   Yes!



7. (10 points) Suppose that a connected planar 3-regular graph has eight vertices. How many regions are created by a planar representation of this graph?
   By the handshaking lemma, a 3-regular graph with 8 vertices will have 12 edges. Euler's formula states $r = e - n + 2$.So, $r = 12 - 8 + 2 = 6$.

8. (20 points) State **Kuratowski's Theorem**.
   A graph G is planar if and only if it contains no subgraphs homeomorphic to $K_{3,3}$ or K_5.
   State the definition of an **elementary subdivision**.
   Let $G = (V, E)$ be a graph with the $a - b$ edge. An elementary subdivision of G using the edge $a - b$ will delete the $a - b$ edge, create an additional vertex c and create the edges $a - c$ and $b - c$.
   State the definition of **Homeomorphic graphs**.
   Two graphs are homeomorphic to each other if one can be derived from elementary subdivisions of the other.
   Give an example of two homeomorphic graphs G_1 and G_2 such that G_1 has 8 edges and G_2 has 11 edges.
   The graphs C_8 and C_{11} form one such pair. The graphs P_9 and P_{12} form another pair. Many examples can be constructed.

9. (15 points) Determine if the following graphs are planar. If a graph is planar, draw a planar representation of it. If not, use Kuratowski's Theorem to show why not.
   i. This graph is non-planar since the vertices $\{2, 3, 5\}$ and $\{1, 4, 6\}$ form a $K_{3,3}$.



   ii. This graph is planar as demonstrated by its planar representation below. Furthermore note that this graph is W_5 and all wheel graphs are planar.