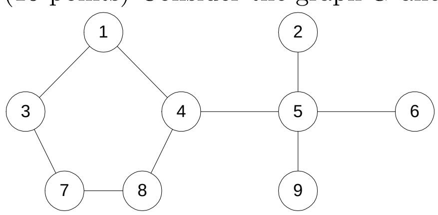
Math 4322 Quiz III
DeMaio Spring 2010

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (15 points) Consider the graph G and compute each of the following.



   i. $\omega(G) = 2$
   ii. $\beta_0(G) = 5$
   iii. $\gamma(G) = 3$

2. (20 points) i. Compute $\beta_0(K_{4,7})$. 7
   ii. Compute the number of different independent sets (of all sizes) in $K_{4,7}$. $1 + 2^4 - 1 + 2^7 - 1 = 143$
   iii. Compute the number of different independent sets (of all sizes) in $K_{m,n}$. $1 + 2^m - 1 + 2^n - 1 = 2^m + 2^n - 1$

3. (20 points) Prove that P_n has F_{n+2} independent sets. First note that P_1 has two independent sets: $\{\emptyset, \{1\}\}$ and P_2 has three independent sets:$\{\emptyset, \{1\}, \{2\}\}$. In other words, P_1 has $F_3 = 2$ independent sets and P_2 has $F_4 = 3$ independent sets. So, the first two values of the Fibonacci sequence line up as we claim they should. Consider P_n. Every independent set of P_n either includes n or does not include n. If n is not in the independent set then we really just have all the independent sets of P_{n-1}.

4. (15 points) Prove $\gamma(R_n) = n$. Keep in mind that such a proof will have two parts!
   First place a rook on every square of the first row. Clearly the board is dominated and $\gamma(R_n) \leq n$. Can the board be dominated with $n-1$ rooks? When placing $n-1$ rooks there will be at least one row without a rook, say row i, and at least one column without a rook in it, say column j. This means that the (i, j) square is not dominated. Hence, $n-1$ rooks are insufficient and $\gamma(R_n) > n-1$. Put the two together and $\gamma(R_n) = n$.

5. (10 points) Dominate a 5×7 board with 3 queens.

Q			Q			Q

6. (10 points) Dominate a 6×8 board with 8 knights.

	N	N			N	N	
	N	N			N	N	

7. (10 points) Provide an example that shows $\beta_0(B_4) > 4$.

B			
B			B
B			B
B			

8. (15 points) i. Give an example of a graph G where $\gamma(G) = \beta_0(G)$. Both C_4 and P_4 have $\gamma(G) = \beta_0(G) = 2$.
ii. Prove $\gamma(G) \leq \beta_0(G)$. Let S be an independent set of size $\beta_0(G)$. The set S must dominate G. If S is not a dominating set then there exists some vertex $v \in V$ such that $v \notin S$ and v is not adjacent to any member of S. If that were true then $S \cup \{v\}$ would be an independent set of size $\beta_0(G) + 1$. This is a blatant contradiction. Thus, S must be a dominating set. This means a dominating set of minimum size has size less than or equal to the size of S. Hence, $\gamma(G) \leq \beta_0(G)$.