Math 3322 Quiz III
DeMaio Spring 2010

Name_____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (20 points) Provide a combinatorial proof that $k\binom{n}{k} = n\binom{n-1}{k-1}$.
   Let $A = \{1, 2, ..., n\}$. Suppose S is the collection of all possible combinations of lottery numbers of size k where one number is the power ball. On the one hand, we can pick the k-set from A in $\binom{n}{k}$ ways and then select the power ball number from that set in k ways. Thus, $|S| = k\binom{n}{k}$. On the other hand we can pick the power ball from A in n ways and then pick the rest of the lottery numbers in $\binom{n-1}{k-1}$ ways. Now, $|S| = n\binom{n-1}{k-1}$. But, we have counted the same set S in two different ways and $k\binom{n}{k} = n\binom{n-1}{k-1}$.

2. (25 points) A donut shop offers 15 different varieties of donuts. How many ways can
   i. John select a dozen different donuts? $\binom{15}{12} = 455$
   ii. John, Paul and George each select one donut? $15^3 = 3375$
   iii. Ringo purchase two dozen donuts for band rehearsal? $\binom{15+24-1}{24} = 9669\,554\,100$
   iv. Ringo purchase two dozen donuts with at least one of each type for band rehearsal? $\binom{15+9-1}{9} = 817\,190$
   v. Ringo purchase two dozen donuts with at least three jelly-filled donuts and no more than two glazed donuts?
   $\binom{14+21-1}{21} + \binom{14+20-1}{20} + \binom{14+19-1}{19} = 1848\,523\,800$

3. (10 points) How many strings of ternary digits (0, 1 or 2)
   i. of length eight exist? $3^8 = 6561$
   ii. of length eight that contain exactly three 0's, two 1's and three 2's? $\frac{8!}{3!2!3!} = 560$

4. (10 points) How many ways are there to choose 18 coins from a piggy bank containing 50 identical nickels, 100 identical dimes and 80 identical quarters? $\binom{3+18-1}{18} = 190$

5. (10 points) What is the probability that a positive integer not exceeding 1000 selected at random is divisible by 9 or 15? $p = \frac{\lfloor \frac{1000}{9} \rfloor + \lfloor \frac{1000}{15} \rfloor - \lfloor \frac{1000}{\mathrm{lcm}(9,15)} \rfloor}{1000} = \frac{31}{200} = 0.155$

6. (15 points) A single card is selected from a standard deck of playing cards. What is the probability that you draw
   i. an *Ace* or a *King*? $p = \frac{4+4}{52} = \frac{2}{13} = 0.153\,85$
   ii. an *Ace* or a *Heart*? $p = \frac{4+13-1}{52} = \frac{4}{13} = 0.307\,69$

7. (15 points) In a five card poker hand, compute the probability of a *Full House*. $p = \frac{13\binom{4}{3}12\binom{4}{2}}{\binom{52}{5}} = \frac{6}{4165} = 1.440\,6 \times 10^{-3}$

8. (15 points) If a deck of cards contains two jokers (one red, one black) that can be any desired card, what is the probability of a *Four of a Kind*? $p = \frac{13\binom{6}{4}48}{\binom{54}{5}} = \frac{8}{2703} = 2.959\,7 \times 10^{-3}$