Math 3322 Quiz III
DeMaio Spring 2010

Name_____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (20 points) Provide a combinatorial proof that $\binom{3n}{2} = 3\binom{n}{2} + 3n^2$.
   Suppose $A = \{1, 2, ..., 3n\}$ and S is the collection of all subsets of size 2 from A. On the one hand, it is immediate that $|S| = \binom{3n}{2}$. On the other hand partition A into $B = \{1, 2, ..., n\}$, $C = \{n+1, n+2, ..., 2n\}$ and $D = \{2n+1, 2n+2, ...3n\}$. How can we select two elements from A relative to B, C and D? We can pick two from the same set in $3\binom{n}{2}$ ways. Or, we can select two sets and pick one element from each in $\binom{3}{2}n^2$ ways. Thus, $|S| = 3\binom{n}{2} + 3n^2$. However, we've counted the same set S in two different ways and $\binom{3n}{2} = 3\binom{n}{2} + 3n^2$.

2. (25 points) A donut shop offers 15 different varieties of donuts. How many ways can
   i. John select a dozen different donuts? $\binom{15}{12} = 455$
   ii. John, Paul and George each select one donut? $15^3 = 3375$
   iii. Ringo purchase two dozen donuts for band rehearsal? $\binom{15+24-1}{24} = 9669\,554\,100$
   iv. Ringo purchase two dozen donuts with at least one of each type for band rehearsal? $\binom{15+9-1}{9} = 817\,190$
   v. Ringo purchase two dozen donuts with at least three jelly-filled donuts and no more than two glazed donuts?
   $\binom{14+21-1}{21} + \binom{14+20-1}{20} + \binom{14+19-1}{19} = 1848\,523\,800$

3. (10 points) How many strings can be made from the letters in
   i. *Aardvark*? $\frac{8!}{3!2!} = 3360$
   ii. *Aardvark* if the three *A*s must be consecutive? $\frac{6!}{2!} = 360$

4. (10 points) How many ways are there to choose 18 coins from a piggy bank containing 50 identical nickels, 100 identical dimes and 80 identical quarters? $\binom{3+18-1}{18} = 190$

5. (10 points) What is the probability that a positive integer not exceeding 1000 selected at random is divisible by 9 or 15?
   $p = \frac{\left\lfloor \frac{1000}{9} \right\rfloor + \left\lfloor \frac{1000}{15} \right\rfloor - \left\lfloor \frac{1000}{\text{lcm}(9,15)} \right\rfloor}{1000} = \frac{31}{200} = 0.155$

6. (15 points) A single card is selected from a standard deck of playing cards. What is the probability that you draw
   i. an *Ace* or a *King*? $p = \frac{4+4}{52} = \frac{2}{13} = 0.153\,85$
   ii. an *Ace* or a *Heart*? $p = \frac{4+13-1}{52} = \frac{4}{13} = 0.307\,69$

7. (15 points) In a five card poker hand, compute the probability of a *Flush*.
   $p = \frac{4\binom{13}{5} - 40}{\binom{52}{5}} = \frac{1277}{649\,740} = 1.965\,4 \times 10^{-3}$

8. (15 points) If a deck of cards contains two jokers (one red, one black) that can be any desired card, what is the probability of a *Royal Flush*?
   $p = \frac{4\binom{7}{5}}{\binom{54}{5}} = \frac{14}{527\,085} = 2.656\,1 \times 10^{-5}$