Math 3322 Quiz III
DeMaio Fall 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (10 points) Give a recursive definition of the set of odd positive integers.
   Let $O_n = O_{n-1} + 2$ for $n \geq 2$ where $O_1 = 1$.

2. (15 points) Consider $g(n) = 3g(n-1) + 4$ where $g(0) = -2$. Compute $g(1)$, $g(2)$, $g(3)$ and $g(4)$.
   $g(1) = 3g(0) + 4 = 3 * -2 + 4 = -2$
   $g(2) = 3g(1) + 4 = 3 * -2 + 4 = -2$
   $g(3) = 3g(2) + 4 = 3 * -2 + 4 = -2$
   $g(4) = 3g(3) + 4 = 3 * -2 + 4 = -2$

3. (15 points) Let A, B and C be sets such that $|A| = |B| = |C| = \aleph_0$. Prove $|A \cup B \cup C| = \aleph_0$.
   Since $|A| = |B| = |C| = \aleph_0$, we can write both A, B and C in an ordered fashion. So we will consider $A = \{a_1, a_2, a_3, ...\}$, $B = \{b_1, b_2, b_3, ...\}$ and $C = \{c_1, c_2, c_3, ...\}$. Now we can write $A \cup B \cup C$ as $\{a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3, ...\}$. This proves that $|A \cup B \cup C| = \aleph_0$.

4. (15 points) A McNugget number is the total number of McDonald's Chicken McNuggets in any number of boxes. The original boxes (prior to the introduction of the Happy Meal-sized nugget boxes) were of 6, 9, and 20 nuggets. The largest non-McNugget number is 43. Prove that any number of McNuggets larger than 43 can be purchased via boxes of 6, 9, and 20 McNuggets.

   Since the smallest number of McNuggets we can purchase is 6, we will need 6 base cases starting at 44.
   $44 = 20 + 4 * 6$
   $45 = 5 * 9$
   $46 = 2 * 20 + 6$
   $47 = 20 + 3 * 9$
   $48 = 8 * 6$
   $49 = 2 * 20 + 9$
   Now, assume that $S(44), S(45), ..., S(n)$ are all true for $n \geq 49$ and show that $S(n+1)$ is also true. Note that $n + 1 = 6 + (n - 5)$. Since $n \geq 49$, $n - 5 \geq 44$ and by the induction hypothesis we can buy exactly $n - 5$ McNuggets. Buy another box of 6 McNuggets and we have purchased exactly $n+1$ McNuggets via boxes of 6, 9, and 20 McNuggets. Thus showing that $S(n+1)$ is true.

5. (15 points) State the complete definition of the Fibonacci numbers and use it to complete the table of values for $F_0, F_1, ..., F_{10}$.
   We define $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$ where $F_0 = 0$ and $F_1 = 1$.

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
0	1	1	2	3	5	8	13	21	34	55

6. (15 points) Complete the table below and conjecture (without proof) a formula for $\sum_{i=1}^{n} F_{2i-1}$.

n	$\sum_{i=1}^{n} F_{2i-1}$	F_{2n}
1	F_1	1
2	$F_1 + F_3 = 1 + 2$	3
3	$F_1 + F_3 + F_5 = 3 + 5$	8
4	$F_1 + F_3 + F_5 + F_7 = 8 + 13$	21
5	$F_1 + F_3 + F_5 + F_7 + F_9 = 21 + 34$	55

7. (15 points) Use induction to prove $\sum_{i=0}^{n} F_i^2 = F_n F_{n+1}$ for $n \in Z^+ \cup \{0\}$.

Show $S(0)$ is true.
$\sum_{i=0}^{n} F_i^2 = F_0^2 = 0^2 = 0$
$F_0 F_1 = 0 * 1 = 0$ and $S(1)$ is true.
Show that if $S(n)$ is true then $S(n+1)$ is also true.
Assume $\sum_{i=0}^{n} F_i^2 = F_n F_{n+1}$ and show $\sum_{i=0}^{n+1} F_i^2 = F_{n+1} F_{n+2}$.
$\sum_{i=0}^{n+1} F_i^2 = \sum_{i=0}^{n} F_i^2 + F_{n+1}^2 = F_n F_{n+1} + F_{n+1}^2 = F_{n+1}\left(F_n + F_{n+1}\right) = F_{n+1} F_{n+2}$.

8. (15 points) Use induction to prove $\frac{4^{n+1}+5^{2n-1}}{21} \in \mathbb{Z}$ for $n \in \mathbb{Z}^+$.
Show that $S(1)$ is true.
$\frac{4^{1+1}+5^{2*1-1}}{21} = 1 \in \mathbb{Z}$.
Show that if $S(n)$ is true then $S(n+1)$ is also true.
Assume $\frac{4^{n+1}+5^{2n-1}}{21} \in \mathbb{Z}$ and show $\frac{4^{n+1+1}+5^{2(n+1)-1}}{21} = \frac{4^{n+2}+5^{2n+1}}{21} \in \mathbb{Z}$.
$\frac{4^{n+2}+5^{2n+1}}{21} = \frac{4*4^{n+1}+25*5^{2n-1}}{21} = \frac{4*4^{n+1}+4*5^{2n-1}+21*5^{2n-1}}{21} = 4*\frac{4^{n+2}+5^{2n+1}}{21} + \frac{21*5^{2n-1}}{21} = 4*int + 5^{2n-1} = int$.