Math 3322 Quiz III Key
DeMaio Spring 2009

Name_____

**Instructions.** Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (10 points) Let $g(0) = 3$ and $g(n+1) = 3g(n) + 7$ for $n \in \mathbb{Z}^+$. Compute $g(1)$, $g(2)$, $g(3)$, $g(4)$ and $g(5)$.
   $g(1) = 3 * 3 + 7 = 16$
   $g(2) = 3 * 16 + 7 = 55$
   $g(3) = 3 * 55 + 7 = 172$
   $g(4) = 3 * 172 + 7 = 523$
   $g(5) = 3 * 523 + 7 = 1576$

2. (10 points) Let $f_n$ be the $n^{th}$ Fibonacci number. Prove $\sum_{k=1}^{n} f_{2k-1} = f_{2n}$ for $n \in \mathbb{Z}^+$.

   1. Show $S(1)$ is true.
   $\sum_{k=1}^{1} f_{2k-1} = f_1 = 1$ and $f_{2*1} = f_2 = 1$ and $S(1)$ is true.

   2. Assume $\sum_{k=1}^{n} f_{2k-1} = f_{2n}$ and show $\sum_{k=1}^{n+1} f_{2k-1} = f_{2(n+1)} = f_{2n+2}$.
   $\sum_{k=1}^{n+1} f_{2k-1} = \sum_{k=1}^{n} f_{2k-1} + f_{2(n+1)-1} = \sum_{k=1}^{n} f_{2k-1} + f_{2n+1} = f_{2n} + f_{2n+1} = f_{2n+2}$

   By 1 and 2, $\sum_{k=1}^{n} f_{2k-1} = f_{2n}$ for $n \in \mathbb{Z}^+$.

3. (5 points) Give a recursive definition of the set of positive integer powers of 3.
   Let $t_1 = 3$ and $t_n = 3t_{n-1}$ for $n \geq 2$.

4. (5 points) How many different passwords exist using four lowercase letters followed by two digits (0-9)?
   $26^4 * 10^2 = 45\,697\,600$

   (5 points) How many different passwords exist using four distinct lowercase letters followed by two distinct digits (0-9)?
   $26 * 25 * 24 * 23 * 10 * 9 = 32\,292\,000$

   (5 points) How many different passwords exist using four distinct lowercase or uppercase letters followed by two distinct digits (0-9)?
   $52 * 51 * 50 * 49 * 10 * 9 = 584\,766\,000$

5. (10 points) In an attempt to raise productivity the CANE corporation is scheduled to publicly flog its six least productive employees. In how many different orders can these six employees be made an example of?
   $6! = 720$

6. (10 points) How many different passwords of seven characters exist where each character may be a lowercase letter of the alphabet and contains at least one vowel (a, e, i, o, u: forget about the y)?
$26^7 - 21^7 = 6230\,721\,635$

7. (10 points) How many bit strings of length 7 start with 0 and end with 10?
$2^4 = 16$

8. (10 points) A drawer contains a dozen brown socks and a dozen black socks. A man takes socks out at random in the dark. How many socks must he take out to be sure that he has
i) at least two of the same color;
3
ii) at least two black socks?
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9. (10 points) At a university of 22,000 students, at least how many must share the same birthday (not including the year)? Don't forget leap years.
$\lceil \frac{22000}{366} \rceil = 61$
How many must have the birthday September 19th?
None!

10. (5 points) Ariel wishes to encode every book in her personal library with a code consisting of an uppercase letter followed by two digits. If Ariel has 1000 books in her library can each book receive a unique code?
There are $26 * 10^2 = 2600$ different unique codes. Yes, Ariel can give each book a unique code.

11. (10 points) How many positive integers not exceeding 3000 are divisible by 12 or 15?
$\lfloor \frac{3000}{12} \rfloor + \lfloor \frac{3000}{15} \rfloor - \lfloor \frac{3000}{60} \rfloor = 400$

12. (10 points) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?
$50 * 99 + 1 = 4951$