Math 3322 Quiz IV
DeMaio Fall 2010

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (5 points each) Complete the following.

   The graph K_{35} has $\underline{\binom{35}{2} = 595}$ edges.

   The graph N_{72} has 0 edges.

   The graph P_{42} has 41 edges.

   The graph C_{112} has 112 edges.

   If the degree sequence of a graph G is $4, 3, 3, 2, 2$ then G has $\underline{7}$ edges.

2. (5 points each) Let $G = (V, E)$ be a graph and $v \in V$. State the definitions of:
   $N(v)$ is the set of all vertices adjacent to v.
   $\deg(v)$ is the cardinality of $N(v)$
   v is an isolated vertex if $\deg(v) = 0$
   v is a pendant if $\deg(v) = 1$

3. (10 points) Use the Binomial Theorem to expand $(2x-3)^4 = 16x^4 - 96x^3 + 216x^2 - 216x + 81$. You must show all details of your work.
   $(2x-3)^4 = \binom{4}{4}(2x)^4(-3)^0 + \binom{4}{3}(2x)^3(-3)^1 + \binom{4}{2}(2x)^2(-3)^2 + \binom{4}{1}(2x)^1(-3)^3 + \binom{4}{0}(2x)^0(-3)^4 = 16x^4 - 4*8*3x^3 + 6*4*9x^2 - 4*2*27x + 81 = 16x^4 - 96x^3 + 216x^2 - 216x + 81$.

4. (10 points) Prove $\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$ for $n \in \mathbb{Z}^+$. The binomial Thereom states that $\sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i} = (x+y)^n$ for all $x, y \in \mathbb{R}$ and $n \in \mathbb{Z}^+$. Let $x = 1$ and $y = -1$ and the binomial theorem reduces down to $\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$

5. (10 points) Can the following scenario occur? There are 95 students who play at least one of football, basket ball and baseball. There are 64 football players, 28 basketball players and 29 baseball players. There are 17 students who play both football and basketball, 13 students who play both football and baseball and 12 students who play both basketball and baseball. Explain. If this were true then we could solve for x, the number of students who play all three sports. So, $95 = 64+28+29-17-13-12+x$, and $x = 16$. However, it cannot be that 16 students play all three sports when only 12 play both basketball and baseball.

6. (10 points) How many different solutions exist to the equation $w + x + y + z = 20$ for $w, x, y, z \in \mathbb{Z}^+$?
   $\binom{4+16-1}{16} = 969$

7. (10 points) Using a standard deck of 52 cards (with no jokers) determine the probability of a full house. $p = \frac{13\binom{4}{3}12\binom{4}{2}}{\binom{52}{5}} = \frac{6}{4165} = 1.4406 \times 10^{-3}$

8. (5 points each) Susan buys an economy pack of fifty pens. The pens are identical except for color. There are ten of each of five different colors.

   (a) How many different ways can Susan select four pens of different colors to take to work? $\binom{5}{4} = 5$

   (b) How many different ways can Susan select ten pens to take to work? $\binom{5+10-1}{10} = 1001$

   (c) How many different ways can Susan select ten pens to take to work with at least one of each color?
   $\binom{5+5-1}{5} = 126$