Math 3322 Quiz IV
DeMaio Spring 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (20 points) Compute each of the following binomial coefficients.
   i. $\binom{10}{3} = 120$
   ii. $\binom{14}{5} + \binom{14}{4} = 3003$
   iii. $\frac{\binom{10}{2}}{\binom{12}{2}} = \frac{15}{22} = 0.681\,82$
   iv. $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 16$
   v. $\frac{\binom{n}{2}}{\binom{n-1}{2}} = \frac{\frac{n(n-1)}{2}}{\frac{(n-1)(n-2)}{2}} = \frac{n(n-1)}{2} * \frac{2}{(n-1)(n-2)} = \frac{n}{n-2}$

2. (10 points) In our twenty person club, we have eight seniors, four juniors, three sophomores and five freshmen. How man ways can we select a five person committee with at least one senior?
   $\binom{20}{5} - \binom{12}{5} = 14\,712$

3. (5 points) A restaurant offers pizza with eight different toppings available. If a large, medium or small pizza with any combination of toppings (but no toppings may be repeated, i.e. no double pepperoni) and any number of toppings can be selected then how many different pizzas with exactly three toppings can be ordered? $3\binom{8}{3} = 168$

4. (15 points) How many different ways can the letters in the following words be arranged?
   i. *house*; $5! = 120$
   ii. *doctor*; $\frac{6!}{2!} = 360$
   iii. *addiction*. $\frac{9!}{2!2!} = 90\,720$

5. (20 points) A social club consists of 10 pairs of twins, some identical, some fraternal. Of these 20 members, 13 are women and 7 are men.
   i. How many ways can this club select a social coordinator and treasurer if these roles must be served by different club members? $20 * 19 = 380$
   ii. How many ways can this club select a social coordinator and treasurer if these roles must be served by club members of opposite gender? $13 * 7 * 2 = 182$
   iii. How many ways can this club form a six person committee to coordinate the club trip to the annual national meeting of twins? Note that each person on this committee is of equal rank and power. $\binom{20}{6} = 38\,760$
   iv. How many ways can this club form a six person committee that consists of three twin pairs to coordinate the club trip to the annual national meeting of twins? Note that each person on this committee is of equal rank and power. $\binom{10}{3} = 120$
   v. How many ways can this club form a six person committee that consists of six club members but no twin pairs to coordinate the club trip to the annual national meeting of twins? Note that each person on this committee is of equal rank and power. $\binom{10}{6}2^6 = 13\,440$

6. (20 points) Use the Binomial Theorem to expand each of the following into polynomial form. You must show all the details of your use of the binomial theorem.
   i. $(3+2x)^4 = 16x^4 + 96x^3 + 216x^2 + 216x + 81$
   ii. $\left(x^2+2\right)^5 = x^{10} + 10x^8 + 40x^6 + 80x^4 + 80x^2 + 32$

7. (15 points) Use the Binomial Theorem to find the coefficient of x^5 in the polynomial expansion of each of the following.
   i. $(2+x)^8$; $\binom{8}{5}x^5 2^3 = 448x^5$
   ii. $(2-x)^{10}$; $\binom{10}{5}(-x)^5 2^5 = -8064x^5$
   iii. $\left(2-x^3\right)^{10}$; The coefficient is 0 since no integer k exists such that $\left(x^3\right)^k = x^{3k} = x^{10}$.

8. (10 points) Use the binomial theorem to prove $\sum\limits_{k=0}^{n} (-1)^k \binom{n}{k} = 0$.

The Binomial Theorem states $\sum\limits_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n$. Let $x = -1$ and $y = 1$ in the binomial theorem. Now,
$\sum\limits_{k=0}^{n} (-1)^k \binom{n}{k} = \sum\limits_{k=0}^{n} \binom{n}{k} (-1)^k 1^{n-k} = \sum\limits_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n = (-1+1)^n = 0^n = 0.$