Key!

Quiz II Math 3322
DeMaio Fall 2008

1. (5 points each) Compute the following:

a) $\lfloor 5.7 \rfloor =$ 5

b) $\lfloor \pi \rfloor =$ 3

c) $\lceil 11.35 \rceil =$ 12

d) $\prod_{k=-3}^{2} k =$ $(-3)(-2)(-1)(0)(1)(2) = 0$

f) $5! =$ 120

g) $\sum_{i=3}^{6} (i-5)^2 =$ $(3-5)^2 + (4-5)^2 + (5-5)^2 + (6-5)^2$
$= (-2)^2 + (-1)^2 + 0^2 + 1^2 = 4 + 1 + 0 + 1 = 6$

h) $\sum_{i=100}^{200} i =$ $\sum_{i=1}^{200} i - \sum_{i=1}^{99} i = \frac{200(201)}{2} - \frac{99(100)}{2} = 100(201) - 50(99)$
$= 20{,}100 - 4950$
$= 15{,}150$

i) $\frac{200!}{197!3!} =$ $\frac{200 \cdot 199 \cdot 198}{6} = 1{,}313{,}400$

2. (10 points) True or False? If true, prove it. If false, provide a counter-example.

$$\lfloor x^2 \rfloor = \lfloor x \rfloor^2$$

False! Let $x = 1.5$. $\lfloor 1.5^2 \rfloor = \lfloor 2.25 \rfloor = 2$
while $\lfloor 1.5 \rfloor^2 = 1^2 = 1$

3. (5 points each) Let $U = \{1, 2, 3, ..., 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$ and $B = \{3, 6, 9\}$.

a) Compute $A \oplus B$.

$A \oplus B = \{2, 3, 4, 8, 9, 10\}$

b) Compute $\overline{A} \cap \overline{B}$.

$\overline{A} = \{1, 3, 5, 7, 9\}$
$\overline{B} = \{1, 2, 4, 5, 7, 8, 10\}$
$\overline{A} \cap \overline{B} = \{1, 5, 7\}$

c) Compute $\overline{A \oplus B}$.

See a) and
$\overline{A \oplus B} = \{1, 5, 6, 7\}$

4. (5 points) Let A be an infinite countable set. By definition $|A| =$ $\aleph_0$ (symbol) and is named aleph-null.

5. (5 points each) Why is f not a function from $\mathbb{R} \to \mathbb{R}$ if

a) $f(x) = \frac{1}{x}$? Since $0 \in \mathbb{R}$ & division by zero is not defined.

b) $f(x) = \sqrt{x}$? Since $-1 \in \mathbb{R}$ and $\sqrt{-1} = i \notin \mathbb{R}$.

c) $f(x) = \pm\sqrt{x^2+1}$? A function is not permitted to map a single x to multiple values of y.

6. (5 points) Let A and B be sets. Describe the strategy for proving $A = B$.

1st show $A \subseteq B$
and then show $B \subseteq A$.
Together this will show that $A = B$.

7. (10 points) Let A and B be sets. Prove $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

I. Show $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.
Let $x \in \overline{A \cap B}$
$\Rightarrow x \notin A \cap B$
$\Rightarrow x \notin A$ or $x \notin B$
if $x \notin A$ then $x \in \overline{A}$ and $x \in \overline{A} \cup \overline{B}$
or if $x \notin B$ then $x \in \overline{B}$ and $x \in \overline{A} \cup \overline{B}$.
In either case $x \in \overline{A} \cup \overline{B}$
and $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

II. Show $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.
Let $x \in \overline{A} \cup \overline{B}$
$\Rightarrow x \notin A$ or $x \notin B$
In either case $x \notin A \cap B$
$\Rightarrow x \in \overline{A \cap B}$.
Thus, $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$.

By I & II
$\overline{A} \cup \overline{B} = \overline{A \cap B}$. ∎

8. (10 points) Let A and B be sets. Prove $A \oplus B = (A - B) \cup (B - A)$

I. Show $A \oplus B \subseteq (A-B) \cup (B-A)$

Let $x \in A \oplus B$.

$\Rightarrow x \in A$ & $x \notin B$ or $x \notin A$ and $x \in B$.

$\Downarrow$ $\Downarrow$

$x \in A - B$ $x \in B - A$

In either case

$x \in (A-B) \cup (B-A)$.

II. Let $x \in (A-B) \cup (B-A)$.

$\Rightarrow x \in A - B$ or $x \in B - A$

if $x \in A - B$

$\Rightarrow x \in A$ & $x \notin B$

$\Rightarrow x \in A \oplus B$

if $x \in B - A$

$\Rightarrow x \in B$ & $x \notin A$

$\Rightarrow x \in A \oplus B$.

In either case

$x \in A \oplus B$.

$\Rightarrow (A-B) \cup (B-A) \subseteq A \oplus B$.

By I & II

$(A-B) \cup (B-A) = A \oplus B$. $\blacksquare$