Math 3322 Quiz IV
DeMaio Fall 2008

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (30 points) How many strings of eight lowercase English letter are there;
   a) if letters can be repeated; $26^8 = 208\,827\,064\,576$
   b) if no letters can be repeated; $26 * 25 * 24 * 23 * 22 * 21 * 20 * 19 = 62\,990\,928\,000$
   c) that start with '**x**' if letters can be repeated; $26^7 = 8031\,810\,176$
   d) that start with '**x**' if no letters can be repeated; $25 * 24 * 23 * 22 * 21 * 20 * 19 = 2422\,728\,000$
   e) that start with '**ma' (in that order)** if letters can be repeated; $26^6 = 308\,915\,776$
   f) that contains the word math '**math' (in that order but not necessarily at the start)** if letters can be repeated? There are only 5 possible places to begin the word math": with letter # 1 through 5. If you start the word "math" on letter #6 through 8 you will not be able to complete the word before running out of room in the string of 8 letters. $5 * 26^4 = 2284\,880$

2. (15 points) How many different functions are there from a set with 5 elements to a set with:
   a) 3 elements; $3^5 = 243$
   b) 5 elements; $5^5 = 3125$
   c) 7 elements? $7^5 = 16\,807$

3. (15 points) How many different **one-to-one** functions are there from a set with 5 elements to a set with:
   a) 3 elements; 0
   b) 5 elements; $5! = 120$
   c) 7 elements? $\frac{7!}{2!} = 2520$

4. (10 points) How many positive integers between 1 and 100 are divisible by 7 or 11? $\lfloor \frac{100}{7} \rfloor + \lfloor \frac{100}{11} \rfloor - \lfloor \frac{100}{77} \rfloor = 22$

5. (10 points) In a group of 45 people, 23 have brown hair and 18 have brown eyes and 6 have both brown hair and brown eyes. How many have either brown hair or brown eyes? $23 + 18 - 6 = 35$

6. (15 points) Let $A = \{a, b, c\}$. There exist $2^3 = 8$ different subsets of A and there are $2^3 = 8$ different bit strings of length three. Construct all elements in each set and explain how the elements correspond to each other.
   A 0 in space i indicates that element i should be excluded from the subset. A 1 in space i indicates that element i should be included from the subset.

Bit strings	$P(A)$
000	$\emptyset$
100	$\{a\}$
010	$\{b\}$
001	$\{c\}$
110	$\{a, b\}$
101	$\{a, c\}$
011	$\{b, c\}$
111	$\{a, b, c\}$

7. (10 points) Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4. Be very explicit with your application of the Pigeonhole Principle. There are four possible remainders upon division by 4: 0, 1, 2 and 3. There are four pigeonholes labeled by these possible remainders. The integers are our 5 pigeons. We place each integer into a pigeonhole corresponding to its remainder after dividing by 4. We have placed 5 pigeons into 4 pigeonholes. Thus, at least one pigeonhole contains at least two pigeons. This shows that at least two of our five integers yield the same remainder after division by 4.

8. (10 points) John has 13 ordinary coins (pennies, nickels, dimes and quarters) in his pocket.
   a) At least how many of the same coin must John have? $\lceil \frac{13}{4} \rceil = 4$
   b)At least how many pennies must John have? 0

9. (5 points) Jason's CD collection consists of five different rock CD's, three different jazz CD's, two different blues CD's, two different classical CD's and a single folk CD. Jason is planning a trip and randomly selects four CD's. How many different ways can this be done? $\binom{13}{4} = 715$