Computation

Compute the following: (3 points each)

1. $10! =$

2. $\frac{500!}{502!} =$

3. $\binom{18}{4} =$

4. $\left\{ \begin{matrix} 4 \\ 2 \end{matrix} \right\} =$

5. $\left[\begin{matrix} 4 \\ 2 \end{matrix} \right] =$

6. $\chi(K_{73})$ =___________;

7. $\chi(K_{45,89})$ =___________;

8. $\chi\left(\overline{K_{n,m}}\right)$ =___________;

9. The number of functions $f: D \to R$ where $D = \{1, 2, 3, 4\}$ and $R = \{a, b, c, d, e, f\}$.

10. The number of onto functions $f: D \to R$ where $D = \{1, 2, 3, 4\}$ and $R = \{a, b, c, d, e, f\}$.

11. The number of 1-1 functions $f: D \to R$ where $D = \{1, 2, 3, 4\}$ and $R = \{a, b, c, d, e, f\}$.

12. The number of 1-1 and onto functions $f: D \to R$ where $D = \{1, 2, 3, 4\}$ and $R = \{a, b, c, d, e, f\}$.

13. If $P(G, x) = x(x-1)^7 - x(x-1)^5(x-2)$ then $\chi(G)$ =___________.

14. $\chi(C_{73})$ =___________;

15. $\chi(C_{176})$ =___________;

Mathematics of the Chessboard

Compute the following. (3 points each)

1. $\gamma(K_7)$ =___________.

2. $\gamma(Q_8)$ =_____________.

3. $\gamma(B_{27})$ =_____________.

4. $\gamma(R_{35})$ =_____________.

5. $\gamma(K_{4,7})$ =___________;

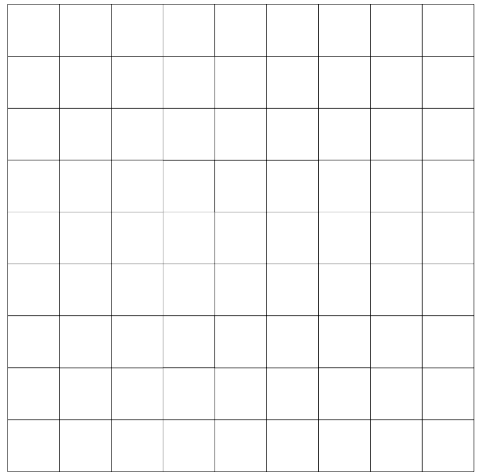
6. $\gamma(R_{7,132})$ =_____________.

7. $\gamma_t(R_{35})$ =_____________.

8. (10 points) Carefully show that a closed knight's tour does not exist on the 3 by 4 board.

1	2	3	4
5	6	7	8
9	10	11	12

9. (10 points) Demonstrate the Spencer-Cockayne construction for dominating the 9×9 chessboard with queens.



Proof

1. (10 points) Use induction to prove $\sum_{i=1}^{n} i^3 = \frac{n^2 * (n+1)^2}{4}$ for all positive integers n.

2. (10 points) Combinatorially prove $\left\{ {n \atop n-2} \right\} = \binom{n}{3} + \dfrac{\binom{n}{2}\binom{n-2}{2}}{2}$.

3. (10 points) Let $G = (V, E)$ be a graph with $n \geq 2$ vertices. Use the pigeonhole principle to prove that there must exist $a, b \in V$ such that $\deg(a) = \deg(b)$.