1. (5 points) How many different ways can the letters in the word *stygian* be arranged?

(5 points) How many different ways can the letters in the word *stygian* be arranged such that the word *sing* appears in the arrangement?

2. (5 points) State the Handshaking Lemma.

(5 points) Use the Handshaking Lemma to show that a graph with 2 vertices of degree 5, 10 vertices of degree 4, 5 vertices of degree 3, 12 vertices of degree 2 and 8 vertices of degree 1 does not exist.

3. (10 points) Do one of the following two induction problems. Clearly indicate your selection.

a. Find and prove the correctness of a formula for $\sum_{i=1}^{n} 2i = 2+4+6+8+...+2n$.

b. Prove $\frac{5^{n+2}+9^{n}+10}{4}$ is an integer for all positive integers n.

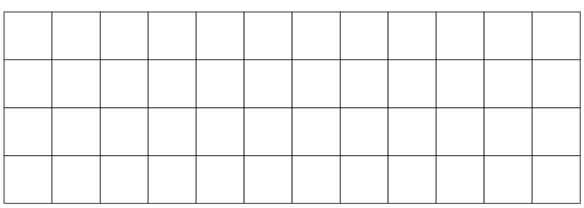
4. Susan buys an economy pack of fifty pens. The pens are identical except for color. There are ten of each of five different colors.

   a. (5 points) How many different ways can Susan select four pens of different colors to take to work?

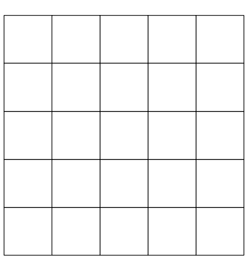
   b. (5 points) How many different ways can Susan select ten pens to take to work?

   c. (5 points) How many different ways can Susan select ten pens to take to work with at least one of each color?

5. (10 points) Dominate the 4 by 12 board with 16 knights.



6. (10 points) Totally dominate the 5 by 5 board with 5 kings.



7. (10 points) Prove that a closed knight's tour does not exist on the 3 by 4 board.

1	2	3	4
5	6	7	8
9	10	11	12

8. (10 points) Find an open knight's tour on the 3 by 4 board that begins at 1 and ends at 12.

1	2	3	4
5	6	7	8
9	10	11	12

9. (10 points) Show $3\binom{n}{2} = 2\binom{n}{3}$ has no integer solutions for $n \geq 2$.

10. Complete the following.

(3 points) The complete graph K_{42} has________________ edges.

(3 points) The complete bipartite graph $K_{42,57}$ has________________ edges.

(3 points) For the complete graph K_n, $\gamma_t(K_n) =$ _____________ .

(3 points) For the cycle graph C_n, $\gamma(C_n) =$ _____________ for $n \geq 3$.

(3 points) For the complete bipartite graph $K_{n,m}$, $\gamma_t(K_{n,m}) =$ ____________ .