Math 4322 Test II
Spring 2007 DeMaio

1. (4 points each) State, without proof, the following values:

a. $\chi(K_{73})$ =___________;

b. $\chi(K_{45,89})$ =___________;

c. $\chi\left(\overline{K_{n,m}}\right)$ =___________;

d. If $P(G,x) = x(x-1)^7 - x(x-1)^5(x-2)$ then $\chi(G)$ =___________;

e. $\gamma(K_7)$ =___________;

f. $\gamma(K_{4,7})$ =___________;

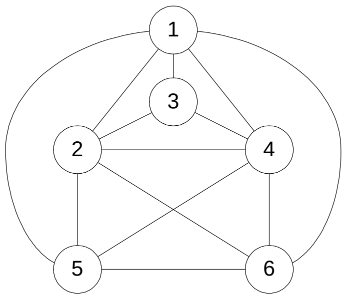
g. $\gamma(Q_5)$ =___________;

h. $\gamma(B_{27})$ =___________;

i. $\gamma(R_{35})$ =___________;

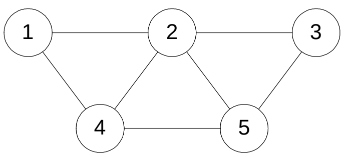
j. $\gamma(R_{7,32})$ =___________.

2. (5 points) Consider the following graph G. Find and prove the correctness of $\chi(G)$.



3. Consider the following graph G.

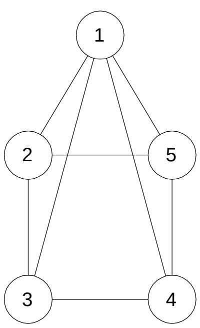
a. (5 points) Find $P(G, x)$.



b. (5 points) With six different colors at your disposal, how many different ways can you color G?

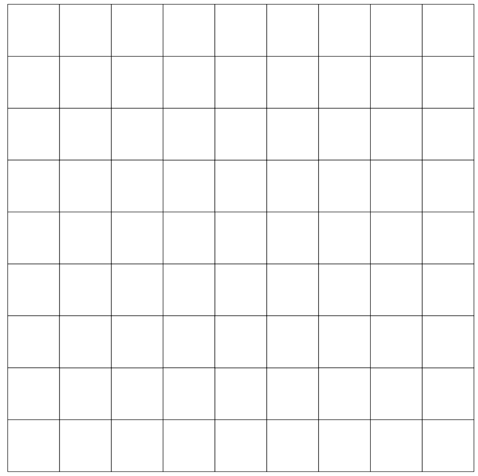
4. Consider the following graph G.

a. (5 points) Find and prove the correctness of $\chi(G)$.

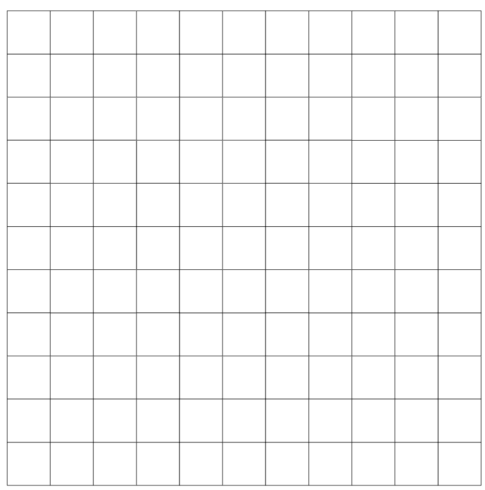


b. (10 points) Find $P(G, x)$.

5. (10 points) Demonstrate the Spencer-Cockayne construction for dominating the 9×9 chessboard with queens.



6. (10 points) Demonstrate the Welch upper bound construction for dominating the 11×11 chessboard with queens.



7. (10 points) Prove $\gamma(B_5) = 5$. Your proof should include (but not be limited to) appropriate pictures.

Bonus! (5 points) An edge-coloring of a graph is an assignment of colors to each edge of the graph. Note that edges incident to the same vertex may receive the same color. Demonstrate by example that it is possible to edge-color K_5 with two colors, red and blue, in such a way that there will be no monochromatic K_3's (i.e. The edges of a subgraph K_3 are either all red or all blue).

Bonus! (10 points) Prove that no matter how you edge-color K_6 with red and blue colors, there will be a monochromatic K_3. Please note that *prove* does not mean *give an example of one case where it works*.