Math 4322 Test III
DeMaio Spring 2008

1. (5 points) Current United States coinage consists of the cent, nickel, dime, quarter, half and dollar. Can John randomly select a certain number of coins from a bank teller (with a theoretically infinite supply of each coin) and be guaranteed at least two of each denomination? If so, how many? If not, why not?

2. (5 points) In a class of 63 students, how many students must be born in the same month?

3. (15 points) A social worker has 77 days to make his rounds. He wants to make at least one visit a day, and has 132 visits to make. Prove there is a period of consecutive days in which he makes exactly 21 visits.

4. (15 points) Prove $\left\{ \begin{matrix} n \\ 2 \end{matrix} \right\} = 2^{n-1} - 1.$

5. (10 points) Compute the number of permutations of the first n integers such that at least one integer is in its natural position.

6. (10 points) Complete the table below for $\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$.

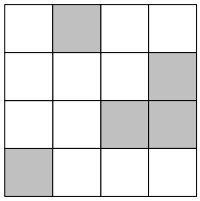
$\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$	1	2	3	4	5	6	7	8	9
1	1								
2	1	1							
3	1	3	1						
4	1	7	6	1					
5	1	15	25	10	1				
6	1	31	90	65	15	1			
7	1	63	301	350	140	21	1		
8	1	127	966	1,701	1,050	266		1	
9	1	255	3,025	7,770	6,951	2,646	462	36	1
10	1	511		34,105	42,525	22,827	5,880	750	
11	1	1,023	28,501	145,750		179,487	63,987	11,880	1,155
12	1	2,047	86,526	611,501	1,379,400	1,323,652	627,396	159,027	22,275

7. (5 points) Compute the number of onto functions $f : D \to R$ where $|D| = 8$ and $|R| = 4$.

8. (10 points) State the recursive formula for d_n. Use it to complete the following table.

n	1	2	3	4	5	6	7
d_n	0	1	2				

9. (15 points) Use rook polynomials to determine the number of arrangements of four non-taking rooks that avoid the shaded cells of the board below.



10. (10 points) Of 200 movie patrons surveyed, 78 always bought candy or popcorn. Of those people, 45 always bought candy while 51 always bought popcorn. How many people always buy both popcorn and candy?

11. (10 points) A small local business has 85 offices. Of these offices, 70 have a computer, 25 have a fax machine and 33 have a paper shredder. There are 20 offices that have both a computer and a fax machine, 27 offices that have both a computer and paper shredder and 15 offices with both a fax machine and paper shredder. There are 12 offices that have a computer, fax machine and paper shredder. How many offices have none of computer, fax machine or shredder?

12. (10 points) Can the following scenario occur? Explain. There are 95 students who play at least one of football, basket ball and baseball. There are 64 football players, 28 basketball players and 29 baseball players. There are 17 students who play both football and basketball, 13 students who play both football and baseball and 12 students who play both basketball and baseball.