Math 3322 Test I
DeMaio Fall 2012

Name____________________

**Instructions.** Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

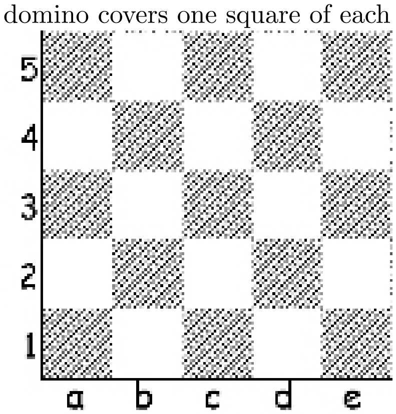
1. (30 points) Compute the following.
   i. $\left\lceil \left(\frac{1}{33}\right)^{250} \right\rceil = 1$
   ii. $\sum_{i=0}^{4} \sum_{j=2}^{4} ij = \sum_{i=0}^{4} 2i + 3i + 4i = \sum_{i=0}^{4} 9i = 0 + 9 + 18 + 27 + 36 = 90$
   iii. For $f$ the function that assigns to each bit string, five times the number of 0's in the bit string, evaluate $f(1011001011)$.
   $f(1011001011) = 5 * 4 = 20$
   iv. $\left\lfloor \left\lceil \frac{5}{4} \right\rceil - \frac{2}{4} \right\rfloor = \left\lfloor 2 - \frac{2}{4} \right\rfloor = \lfloor 1.5 \rfloor = 1$
   v. $\frac{100!}{98!} = \frac{100*99*98!}{98!} = 100 * 99 = 9900$
   vi. $\prod_{k=1}^{25} \left(k^2 - 81\right) = 0$ since $k = 9$ will yield a zero in the product.

2. (5 points) Why is $f : \mathbb{Z} \to \mathbb{R}$ via $f(x) = \dfrac{1}{\sqrt{x + \frac{1}{2}}}$ not a function? Because for any negative integer $x$, the function yields a complex number which is not in the codomain $\mathbb{R}$. Note that $x = -\frac{1}{2}$ in not a problem since $-\frac{1}{2} \notin \mathbb{Z}$.

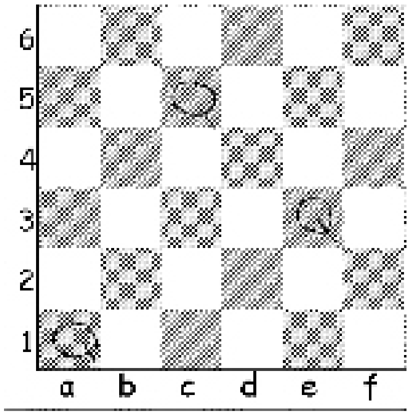
3. (10 points) Give an example of a function $f : \mathbb{Z}^+ \to \mathbb{Z}^-$ that is neither one-to-one nor onto. Explain why your example is correct. Be certain that your example is a function!
   Let $f(x) = -13$. Clearly, $f$ is not onto since there is no pre-image of $-5$. Equally clearly, $f$ is not onto since $f(5) = f(6) = -13$.

4. (10 points) Let $f$ be the function that assigns to each bit string the quotient of the number of 0s in the string divided by the number of 1s in the string. State the domain and range of $f$.
   The domain is the collection of all bit strings with at least a single 1. The range is the collection of all non-negative rational numbers.

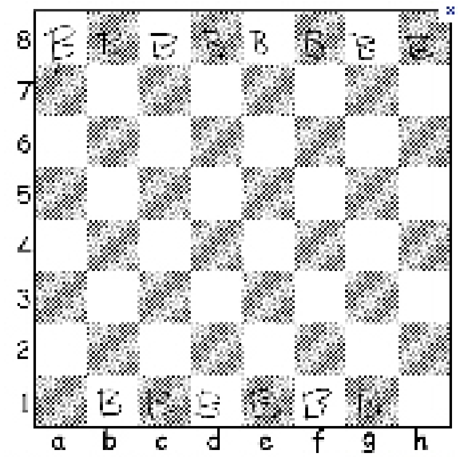
5. (10 points) Can you tile with dominoes the $5 \times 5$ board after removing three corner squares? If yes, do so. If not, explain why not.
   No. After removing three corner squares there will be 10 black squares and 12 white squares. Since a domino covers one square of each color, it is impossible to tile this board.




6. (10 points) Dominate the $6 \times 6$ chessboard with 3 queens.



7. (10 points) Place 14 independent bishops on the $8 \times 8$ board.



8. (10 points) Find terms $a_{30}, a_{31}, a_{32}, a_{33}, a_{34}$ in the sequence where the $n^{th}$ term of the sequence is the number of bits in the binary expansion of $n$.

| decimal $n$ | binary $n$ | $a_n$ |
|---|---|---|
| 30 | 11110 | 5 |
| 31 | 11111 | 5 |
| 32 | 100000 | 6 |
| 33 | 100001 | 6 |
| 34 | 100010 | 6 |

9. (15 points) Use induction to prove $\sum_{k=1}^{n}(2k-1) = n^2$ for $n \in \mathbb{Z}^+$.

First, show $S(1)$ is true.

L.H.S. $\sum_{k=1}^{1}(2k-1) = 2*1-1 = 1$

R.H.S. $1^2 = 1$

Now show that if $S(n)$ is true then $S(n+1)$ is also true. So, assume $\sum_{k=1}^{n}(2k-1) = n^2$ and show $\sum_{k=1}^{n+1}(2k-1) = (n+1)^2$.

So, $\sum_{k=1}^{n+1}(2k-1) = \left(\sum_{k=1}^{n}(2k-1)\right) + (2(n+1)-1)$. Using the inductive assumption this becomes $n^2 + (2(n+1)-1) = n^2 + 2n + 1 = (n+1)^2$.