Math 3322 Test I
DeMaio Spring 2013

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (25 points) Compute the following.
   i. $\left\lfloor \left(-\frac{1}{33}\right)^{255} \right\rfloor = -1$
   ii. $\left\lceil \frac{1000!}{2000!} \right\rceil = 1$
   iii. Let f be the function that assigns to each bit string the number of 0s in the bit string minus the number of 1s in the bit string. $f(1011001011) = 4 - 6 = -2$.
   iv. $\left\lfloor \left\lceil \frac{5}{4} \right\rceil - \frac{2}{4} \right\rfloor = 1$
   v. $\frac{100!}{98!} = 9900$

2. (10 points) Why is $f : \mathbb{Z} \to \mathbb{R}$ via $f(x) = \dfrac{1}{\sqrt{x + \frac{1}{2}}}$ not a function? The square root of a negative number is not real. So, $f(-5)$ is undefined.

3. (10 points) Why is $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+$ via $f(m, n) = m^2 + n^2$ not a 1-1 function? Since $f(1, 0) = f(0, 1) = 1$ but $(1, 0) \neq (0, 1)$.

4. (10 points) Let f : set of all bitstrings $\to \mathbb{Z}$ be the function where f assigns to each bit string the number of 0s in the bit string minus the number of 1s in the bit string.
   Is f 1-1? Explain. No. Since $f(10) = f(01) = 0$ but $10 \neq 01$.
   Is f onto? Explain. Yes. Note that $f(01) = 0$. If d is a positive integer then the bit string of d 0s and no 1s maps to d.If d is a negative integer then the bit string of d 1s and no 0s maps to d.

5. (10 points) Give an example of a function $f : \mathbb{Z}^+ \to \mathbb{Z}^-$ that is neither one-to-one nor onto. Explain why your example is correct. Be certain that your example is a function! Let $f(x) = -42$. This function is not 1-1 since $f(1) = f(2) = -42$ but $1 \neq 2$. This function is not onto since -5 has no pre-image.

6. (10 points) Give an example of a function $f : \mathbb{Z}^+ \to \mathbb{Z}^-$ that is both one-to-one and onto. Explain why your example is correct. Be certain that your example is a function! Let $g(x) = -x$. Pick any negative integer x. Note that $-x$ is positive and $f(-x) = - - x = x$. It is 1-1 since if $-x = -y$ then $x = y$.

7. (5 points) Let f be the function that assigns to each bit string the product of the number of 0s in the string and the number of 1s in the string. State the range of f. $\mathbb{Z}^+ \cup \{0\}$

8. (10 points) Can you tile the 5×7 board with dominoes after removing any three corner squares? If yes, do so. If not, explain why not. No. After removing three corner squares the board will contain 17 white squares and 15 black squares. Since a domino covers one square of each color, it is impossible to tile this board.

9. (10 points) Can you tile the 5×7 board with right trominoes after removing any three corner squares? If yes, do so. If not, explain why not. No. After removing three corner squares the board will contain 32 squares which is not divisible by 3.

10. (10 points) Give an example of a (mutilated or otherwise) 5×5 board where every square has at least one match and the number of black squares is equal to the number of white squares yet the board cannot be tiled with dominoes. Explain why your example works. Remove all squares in the second row and any two black squares below the second row. It is now impossible to tile the top row of three black squares and two white squares since a domino covers one square of each color.