Math 3322 Test I
DeMaio Spring 2011

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (10 points) Let $S = \{x | x \in \mathbb{Z}^+, x \text{ is a solution of } \left(x^2 - 4\right)(2x + 7) = 0\}$. List the elements of S. $S = \{2\}$.
2. (10 points) Use set builder notation to define the set of all rational numbers. $\mathbb{Q} = \{\frac{a}{b} | a, b \in \mathbb{Z}, b \neq 0\}$
3. (10 points) Let $A = \{1, \{1\}\}$. Construct $P(A)$. $P(A) = \{\emptyset, \{1\}, \{\{1\}\}, \{1, \{1\}\}\}$
4. (15 points) Compute the cardinality of each of the following sets.
   i. $\emptyset$; $|\emptyset| = 0$
   ii. $B = \{1, 2, 3, \{\alpha, \omega, 1\}, \beta, c, \{1\}, \{\{\alpha, \beta, \gamma, \delta\}\}, \mathbb{Z}, \mathbb{Z}^+, \mathbb{R}\}$; $|B| = 11$
   iii. $P(C)$ for $C = \{1, 2, 3, ..., 10\}$. $|P(C)| = 2^{10} = 1024$
5. (35 points) Let $S = \{\{1\}, \{3\}, 2, 3, \emptyset\}$. Answer the following without explanation.
   i. Is $1 \subseteq S$? No.
   ii. Is $\{2, 3\} \subseteq S$? Yes.
   iii. Is $\{1, 3\} \in S$? No.
   iv. Is $\emptyset \in S$? Yes.
   v. Is $\{\emptyset\} \in S$? No.
   vi. Is $\{3, \{2, 3\}\} \subseteq S$? No.
   vii. Is $(3, \{3\}) \in S \times S$?Yes
6. Let A and B be sets.
   i. (5 points) State the definition of A is a **subset** of B. A is a **subset** of B if every element in A is also an element in B.
   ii. (5 points) What is the difference in meaning of $A \subseteq B$ versus $A \subset B$? In $A \subseteq B$, we allow for the possibility that $A = B$. In $A \subset B$. we know that $A \neq B$.
   iii. (10 points) Give an example of two sets A and B such that $A \in B$ and $A \subseteq B$. Many examples exist. Let $A = \{1\}$ and $B = \{1, \{1\}\}$.
7. (10 points) Let A and B be nonempty sets such that $A \neq B$. Prove $A \times B \neq B \times A$. If $A \neq B$ then there exists (without loss of generality) $x \in A$ such that $x \notin B$. Let $y \in B$. Note that $(x, y) \in A \times B$ but since $x \notin B$ it is clear that $(x, y) \notin B \times A$.