Math 3322 Test II
DeMaio Fall 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (10 points) A certain shirt is produced for both men and women. How many different shirts exist if
i. the shirt comes in 3 sizes and 5 colors for both men and women; $2*3*5=30$
ii. the shirt comes in 3 sizes and 12 colors for women and 5 sizes and 6 colors for men? $3*12+5*6=66$

2. (15 points) How many ways can we rearrange the letters in the word
i. vampire; $7!=5040$
ii. werewolf; $\frac{8!}{2!*2!}=10\,080$
iii. mummy? $\frac{5!}{3!}=20$

3. (5 points) In an attempt to raise productivity the CANE corporation is scheduled to publicly flog its six least productive employees. In how many different orders can these six employees be flogged? $6!=720$

4. Let $S=\{0,1,2,3,4,5,6,7\}$.
i. (5 points) How many non-empty subsets does S have? $2^8-1=255$
ii. (5 points) How many subsets of S have no odd numbers as members? $2^4=16$
iii. (5 points) How many subsets of S have exactly 4 elements? $\binom{8}{4}=70$
iv. (5 points) How many subsets of S have an odd number elements? $\binom{8}{1}+\binom{8}{3}+\binom{8}{5}+\binom{8}{7}=128$
v. (5 points) How many four digit numbers can be made using the digits of S if a digit may be used repeatedly? Before you answer, ask yourself if 0 can be a leading digit. $7*8^3=3584$
vi. (10 points) How many even, four digit numbers can be made using the digits of S if a digit may be used only once? **Be careful!** There is a reason this part is worth 10 points. If we pick the last digit first, we run into a problem picking the first digit since we do not know if we used zero or not for the final digit. This needs to be broken into two cases; the last digit is zero or the last digit is not zero. We have $1*7*6*5+3*6*6*5=750$ different even four digit numbers.

5. (5 points) How many positive integers not exceeding 3000 are divisible by 12 or 15? $\left\lfloor\frac{3000}{12}\right\rfloor+\left\lfloor\frac{3000}{15}\right\rfloor-\left\lfloor\frac{3000}{\operatorname{lcm}(12,15)}\right\rfloor=400$

6. (5 points) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state? $99*50+1=4951$

7. (5 points) At a university of 22,000 students, at least how many must share the same birthday (not including the year)? Don't forget leap years. $\left\lceil\frac{22000}{366}\right\rceil=61$
(5 points) How many must have the birthday September 19th? None.

8. (10 points) Use the Binomial Theorem to expand $(3x-2)^4$ into standard polynomial form. You must show all the details of your use of the Binomial Theorem.
$\binom{4}{4}(3x)^4(-2)^0+\binom{4}{3}(3x)^3(-2)^1+\binom{4}{2}(3x)^2(-2)^2+\binom{4}{1}(3x)^1(-2)^3+\binom{4}{0}(3x)^0(-2)^4=81x^4-216x^3+216x^2-96x+16$

9. (15 points) Use the Binomial Theorem to find the coefficient of x^8 in the expansion of each of the following. Once again, you must show all the details of your use of the Binomial Theorem.
i. $(2x-3)^{10}$; $\binom{10}{8}(2x)^8(-3)^2=103\,680x^8$
ii. $\left(5x^3-6\right)^9$; 0
iii. $\left(5x^4-3\right)^5$. $\binom{5}{2}\left(5x^4\right)^2(-3)^3=-6750x^8$

10. (10 points) Use the Binomial Theorem to prove $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$. Let $x = -1$ and $y = 1$. Thus, $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = \sum_{k=0}^{n} (-1)^k * 1^{n-k} \binom{n}{k} = (-1+1)^n = 0^n = 0$.