Math 3322 Test II

DeMaio Spring 2011

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (35 points) Compute the following.
   i. $\left\lceil \left(\frac{1}{33}\right)^{250} \right\rceil = 1$
   ii. $\lceil \pi \rceil - \lfloor e \rfloor = 2$
   iii. $\frac{250!}{3!247!} = \frac{250*249*248}{6} = 2573\,000$
   iv. $\sum_{i=1}^{350} i = 61\,425$
   v. $\sum_{i=1}^{200} i^2 = 2686\,700$
   vi. $\sum_{i=300}^{500} i = \sum_{i=1}^{500} i - \sum_{i=1}^{299} i = 80\,400$
   vii. $\prod_{k=1}^{100} k^{\left\lfloor \frac{1}{k^2} \right\rfloor} = 1$

2. (10 points) Let $A = \{1, 3, 5, 7, 9\}$, $B = \{5, 7, 13\}$ and $U = \{1, 2, 3, ..., 15\}$. Compute the following.
   i. $A \cup B = \{1, 3, 5, 7, 9, 13\}$
   ii. $A \cap B = \{5, 7\}$
   iii. $A \oplus B = \{1, 3, 9, 13\}$
   iv. $\bar{A} \cap \bar{B} = \{2, 4, 6, 8, 10, 11, 12, 14, 15\}$

3. (10 points) Find the first 10 terms of the sequence whose n^{th} term is the smallest integer k such that $k^2 \geq n$.

n	1	2	3	4	5	6	7	8	9	10
a_n	1	2	2	2	3	3	3	3	3	4

4. (10 points) Show that the union of three infinitely countable sets is countable.
   Let A, B, C all have cardinality $\aleph_0$. Since A, B, C are all countable infinite we can write their elements in order. So, $A = \{a_1, a_2, a_3, ...\}$, $B = \{b_1, b_2, b_3, ...\}$ and $C = \{c_1, c_2, c_3, ...\}$. It is now easy to write all the elements of $A \cup B \cup C$ in order as $\{a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3, ...\}$.

5. (15 points) Someone claims that $r_1, r_2, r_3, ...$ is an ordered listing of all reals in $(0, 1)$. Produce $r \in (0, 1)$ such that $r \neq r_i$ for all $i \in \mathbb{Z}^+$.
   See class notes.

6. (10 points) Let f be the function that assigns to each bit string, three times the number of 0's in the bit string. State the domain and range of f.
   The domain is the collection of all bit strings. The range is $\{0, 3, 6, 9, 12, ...\}$.

7. (10 points) True or False? $\lfloor x \lceil y \rceil \rfloor = xy$ for $x, y \in \mathbb{Z}$. If true, prove it. If false, provide a counter example.
   This is true since x and y are integers. Both floor and ceiling leave integers unchanged.

8. (5 points) Let A be the set of students who live within one mile of campus and let B be the set of students who walk to campus. Describe the students in the set $A - B$.
   The set $A - B$ consists of the students who live within one mile of campus but do not walk to campus.

9. (10 points) Give an example of a function $f : \mathbb{Z}^+ \to \mathbb{Z}^-$ that is neither one-to-one nor onto. Explain why your example is correct. The constant function $f(n) = -5$ is one possible example. Since every value is mapped to -5, $f(n)$ is obviously not one-to-one. Since -5 is not the only element in the codomain, $f(n)$ is just as clearly not onto.