Math 3322 Test III
DeMaio Fall 2009

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. **Where appropriate, simplify all answers to a single decimal expansion.**

1. (15 points) i. Consider the equation $x + y + z = 15$. If x, y and z are all non-negative integers, how many different solutions exist? Keep in mind that the solution $x = 3$, $y = 4$ and $z = 8$ is different from the solution $x = 3$, $y = 8$ and $z = 4$. $\binom{3+15-1}{15} = 136$
   ii. Consider the equation $x + y + z = 15$. If x, y and z are all positive integers, how many different solutions exist? $\binom{3+12-1}{12} = 91$

2. (25 points) Susan buys an economy pack of fifty pens. The pens are identical except for color. There are ten of each of five different colors.
   i. How many different ways can Susan select four pens of different colors to take to work? $\binom{5}{4} = 5$
   ii. How many different ways can Susan select ten pens to take to work? $\binom{5+10-1}{10} = 1001$
   iii. How many different ways can Susan select twelve pens to take to work with at least one of each color pen? $\binom{7+5-1}{7} = 330$
   iv. How many different ways can Susan select six pens to take to work with at least two different colors in the mix? $\binom{5+6-1}{6} - 5 = 205$
   v. How many different ways can Susan select twelve pens to take to work? $\binom{5+12-1}{12} - 5 - 5 * 4 = 1795$

3. (5 points) Shot in the back while playing poker, Wild Bill Hickok's final hand was a pair of aces and a pair of eights, now known as the dead man's hand. How many different dead man's hands exist? $\binom{4}{2}^2 * 44 = 1584$

4. (10 points) Compute the probability of a *pair* in the game of poker. $13\binom{4}{2}\binom{12}{3}4^3 = 1098\,240$ different pair hands exist. So the probability of a pair is $\frac{13\binom{4}{2}\binom{12}{3}4^3}{\binom{52}{5}} = \frac{352}{833} = 0.422\,57$

5. (10 points) If a deck of cards contains the two jokers that can be any desired card, which hand should win: a five of a kind or a royal flush? Explain your reasoning and include all necessary combinatorial computations that support your answer.
   Five of a Kind: $13\binom{6}{5} = 78$ different hands.
   Royal Flush: $4\binom{7}{5} = 84$ different hands.
   The five of a kind is a scarcer hand and should win over the royal flush.

6. (10 points) How many positive integers not exceeding 1000 are divisible by 6, 9 or 15?
   $\left\lfloor \frac{1000}{6} \right\rfloor + \left\lfloor \frac{1000}{9} \right\rfloor + \left\lfloor \frac{1000}{15} \right\rfloor - \left\lfloor \frac{1000}{\operatorname{lcm}(6,9)} \right\rfloor - \left\lfloor \frac{1000}{\operatorname{lcm}(6,15)} \right\rfloor - \left\lfloor \frac{1000}{\operatorname{lcm}(9,15)} \right\rfloor + \left\lfloor \frac{1000}{\operatorname{lcm}(6,9,15)} \right\rfloor = \left\lfloor \frac{1000}{6} \right\rfloor + \left\lfloor \frac{1000}{9} \right\rfloor + \left\lfloor \frac{1000}{15} \right\rfloor - \left\lfloor \frac{1000}{18} \right\rfloor - \left\lfloor \frac{1000}{30} \right\rfloor - \left\lfloor \frac{1000}{45} \right\rfloor + \left\lfloor \frac{1000}{90} \right\rfloor = 244$

7. (10 points) Can the following scenario occur? Explain and include all necessary supporting computations. There are 95 students who play at least one of football, basket ball and baseball. There are 64 football players, 28 basketball players and 29 baseball players. There are 17 students who play both football and basketball, 13 students who play both football and baseball and 12 students who play both basketball and baseball.
   Let x be the number of students who play all three sports. If the above scenario is true then we know by the principle of inclusion/exclusion that $95 = 64 + 28 + 29 - 17 - 13 - 12 + x$. Solving this yields $x = 16$. So we have 16 students that play all three sports. This is a problem since only 13 students play football and baseball.

8. (15 points) Six men and seven women stand in a row for a yearbook photo.
   (a) How many arrangements of these people are possible? $13! = 6227\,020\,800$
   (b) How many arrangements of these people are possible if Bruce and Betty must stand beside each other? $12! * 2 = 958\,003\,200$

(c) How many arrangements of these people are possible if Bruce and Betty must not stand beside each other? $13! - 12! * 2 = 5269\,017\,600$

9. (15 points) A woman has 11 close friends. Find the number of ways she can invite 5 of them to dinner where:
i. There are no restrictions. $\binom{11}{5} = 462$
ii. Two of the friends are newlyweds and will not attend separately.
There are two disjoint cases: the couple is invited or not. $\binom{9}{3} + \binom{9}{5} = 210$
iii. Two of the friends are divorced from each other and will not attend together. Here we have two cases: neither are invited or exactly one of the two is invited. $\binom{9}{5} + 2\binom{9}{4} = 378$