Math 3322 Test I
DeMaio Fall 2008

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

Preliminaries

1. (5 points) List the members of the set $S = \left\{x|x \in Z^+, 50 \leq x^3 \leq 150\right\}$.

2. (5 points) Construct $P(A)$ for $A = \{*, a, 3\}$.

3. (5 points) Compute $|P(A)|$ for $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31\}$.

4. (5 points) Give an example of sets A and B such that B is a proper subset of A and $|A| = |B|$.

5. (5 points) In a Venn diagram, shade $\overline{A \oplus B}$.

6. (5 points) Let A be the set of students who live within one mile of campus. Let B be the set of all students who walk to class. What does it mean to say Mary is a member of the set $\overline{A} \cap B$?

7. (5 points) Compute $\left\lfloor \frac{1}{2} + \left\lfloor \frac{1}{2} + \left\lceil \frac{1}{2} \right\rceil \right\rfloor \right\rfloor$.

8. (5 points) Find the domain and range of the function that assigns to each positive integer its last digit.

9. (5 points) Give an example of a function $f : Z \to Z^+$ that is neither one-to-one nor onto.

10. (5 points) Compute $\sum_{i=50}^{175} i$.

11. (5 points) Compute $\prod_{-533}^{278} \left(i^3 - 1\right)$.

12. (5 points) Compute $\frac{100!}{95!5!}$.

Problems

13. (10 points) True or False? If true, prove it. If false, provide a counter-example. $(j+k)! = j! + k!$

14. (10 points) Prove $|Q^+| = \aleph_0$.

15. (10 points) Use mathematical induction to prove $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ for all $n \in Z^+$.

16. (10 points) Use induction to prove 3 divides $n^3 + 2n - 15$ for all $n \in Z^+$.

17. (10 points) Find the error in the following proof of this "theorem":

"Theorem: Every positive integer equals the next largest positive integer."

" Proof: Let $P(n)$ be the proposition '$n = n+1$'. To show that $P(k) = P(k+1)$, assume that $P(k)$ is true for some k, so that $k = k+1$. Add 1 to both sides of this equation to obtain $k+1 = k+2$, which is $P(k+1)$. Therefore $P(k) = P(k+1)$ is true. Hence $P(n)$ is true for all positive integers n.".

18. (10 points) For sets A, B and C, let $A \oplus C = B \oplus C$. Use contradiction to prove $A = B$.