Math 3322 Test III
DeMaio Fall 2008

Name____

Instructions. Show all your work. Credit cannot and will not be awarded for work not shown. Where appropriate, simplify all answers to a single decimal expansion.

1. (4 points each) Complete the following.

   The graph K_{57} has $\binom{57}{2} = 1596$ edges.

   The graph N_{63} has 0 edges.

   The graph P_{47} has 46 edges.

   The graph C_{103} has 103 edges.

   The graph W_{103} has 206 edges.

   The graph W_{103} has 104 vertices.

   The graph $K_{15,17}$ has $15 * 17 = 255$ edges.

   The graph $K_{15,17}$ has $15 + 17 = 32$ vertices.

   The graph K_{35} is regular of degree 34.

   If G is a simple graph with $e = 15$ edges and $\overline{G}$ has $e = 13$ edges then G has $n =$ 8 vertices. since $\binom{8}{2} = 28$.

2. (5 points) Construct a graph $G = (V, E)$ with $n = 6$ vertices and $e = 9$ edges such that $\deg(v) \leq 3$ for all $v \in V$. The graph $K_{3,3}$ satisfies the requirements.

3. (10 points) Suppose a graph has 150 edges, 30 vertices of degree 4, and all others of degree 3. How many vertices does the graph have? Since $\sum_{v \in V} \deg(v) = 2e$ we know that $\sum_{v \in V} \deg(v) = 4*30+3x = 300$ where x is the number of vertices of degree 3. Solving for x, yields $x = 60$. Thus, there are 90 vertices in the graph.

4. (10 points) At a local high school, 50 students are on the football team, 19 on the basketball team, and 25 on the baseball team. There are 12 students who play both football and basketball, 18 who play both football and baseball and 7 who play both basketball and baseball. There are 4 students who play all three sports. How many students play on at least one of football, basketball or baseball? $50 + 19 + 25 - 12 - 18 - 7 + 4 = 61$

5. (10 points) Can the following scenario occur? Explain. There are 95 students who play at least one of football, basket ball and baseball. There are 64 football players, 28 basketball players and 29 baseball players. There are 17 students who play both football and basketball, 13 students who play both football and baseball and 12 students who play both basketball and baseball. No! If it could then $95 = 64+28+29-17-13-12+x$ where x is the number of students who play all 3 sports. Solving for x yields $x = 16$. But, this cannot be the case since only 12 students play both baseball and basketball.

6. (10 points) Use the Principle of Inclusion-Exclusion to find the number of positive integers not exceeding 1000 that are neither the square nor the cube of an integer. There are 1000 positive integers that do not exceed 1000. Since $\sqrt[2]{1000} = 31.623$ there are 31 integers not exceeding 1000 that are squares. Since $\sqrt[3]{1000} = 10$ there are 10 integers not exceeding 1000 that are cubes. However, 1, 64 and 729 are both squares and cubes. Thus, there exist $31 + 10 - 3 = 38$ integers that are squares or cubes. Hence there are $1000 - 38 = 962$ integers not exceeding 1000 that are neither squares nor cubes.

7. (10 points) Shot in the back while playing poker, Wild Bill Hickok's final hand was a pair of aces and a pair of eights, now known as the dead man's hand. How many different dead man's hands exist? $\frac{\binom{4}{2}^2 * 44}{\binom{52}{5}} = \frac{33}{54\,145} = 6.094\,7 \times 10^{-4}$

8. (10 points) Using a standard deck of 52 cards (with no jokers) determine the probability of a full house.

   $\frac{13\binom{4}{3}12\binom{4}{2}}{\binom{52}{5}} = \frac{6}{4165} = 1.440\,6 \times 10^{-3}$

9. (15 points) Using a standard deck of 52 cards (with no jokers) where each of the four deuces is considered a wild card, which hand should win: a five of a kind or a royal flush?

   i) The five of a kind can be made with 1, 2, 3 or 4 deuces. Also note that you cannot achieve a five of a kind with deuces. Using these four disjoint cases, there exist

   $12\binom{4}{4}\binom{4}{1} + 12\binom{4}{3}\binom{4}{2} + 12\binom{4}{2}\binom{4}{3} + 12\binom{4}{1}\binom{4}{4} = 672$ different five of a kinds.

   ii) The royal flush can be made with 0, 1, 2, 3 or 4 deuces.

   $4\binom{5}{5}\binom{4}{0} + 4\binom{5}{4}\binom{4}{1} + 4\binom{5}{3}\binom{4}{2} + 4\binom{5}{2}\binom{4}{3} + 4\binom{5}{1}\binom{4}{4} = 504$ different royal flush hands.

   Since the royal flush is the scarcer hand, it should be the more powerful hand.