Vertex-Edge Graphs

Vertex-Edge Graphs

In the

Georgia Performance Standards

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Math III

- **MM3A7. Students will understand and apply matrix representations of vertex-edge graphs.**
- a. Use graphs to represent realistic situations.
- b. Use matrices to represent graphs, and solve problems that can be represented by graphs.

Vertex-Edge Graphs

- Definitions
- Models
- Examples
- Matrices

- Families
- Euler circuits
- Matchings
- Hamilton cycles

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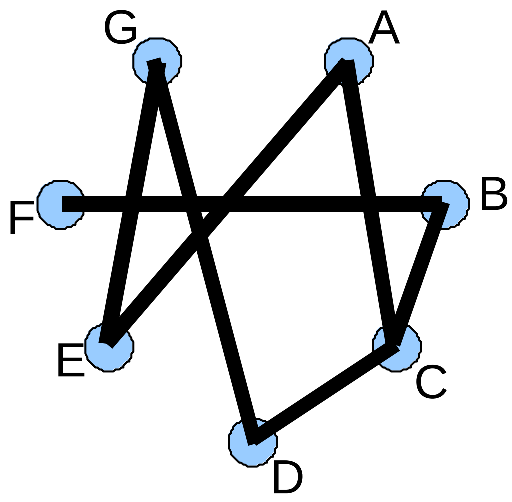
Definition

- Set-theoretically, a graph G is a set of vertices V(G), and a set of unordered pairs of elements of V(G), called E(G), the edges.
- Other definitions for a graph include language (a story problem), visual representations (of dots and lines), integers (operations modulo n), and matrices.

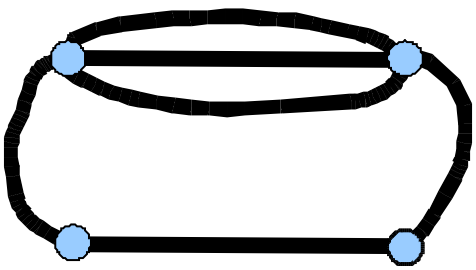
Example

- V(G)={Alan, Bob, Chad, Dave, Ed, Fred, George}
- E(G)={(Alan, Chad),(Dave, George), (Ed, Alan), (Bob, Fred), (Bob, Chad), (Chad, Dave), (George, Ed)}



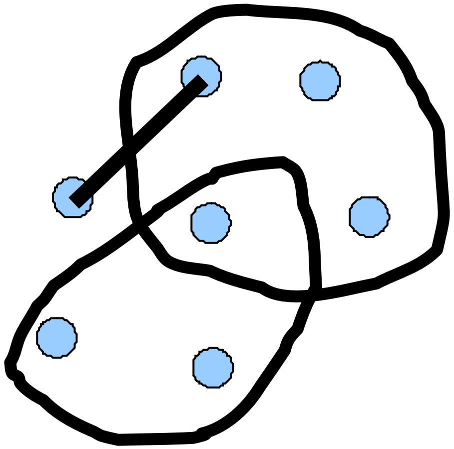
Subtleties of the definitions

- Single edges vs. multiple edges



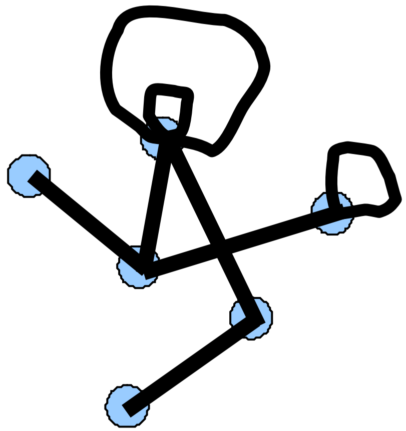
Subtleties of the definitions

- Single edges vs. multiple edges
- Simple edges vs hyperedges



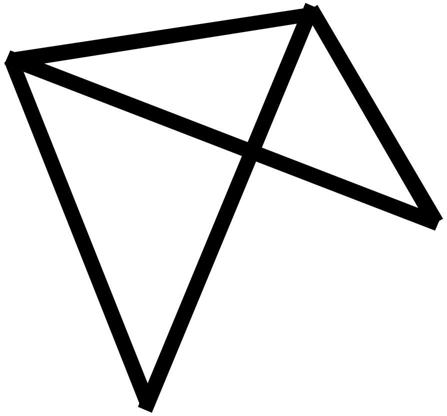
Subtleties of the definitions

- Single edges vs. multiple edges
- Simple edges vs hyperedges
- Simple loopless graphs vs loops

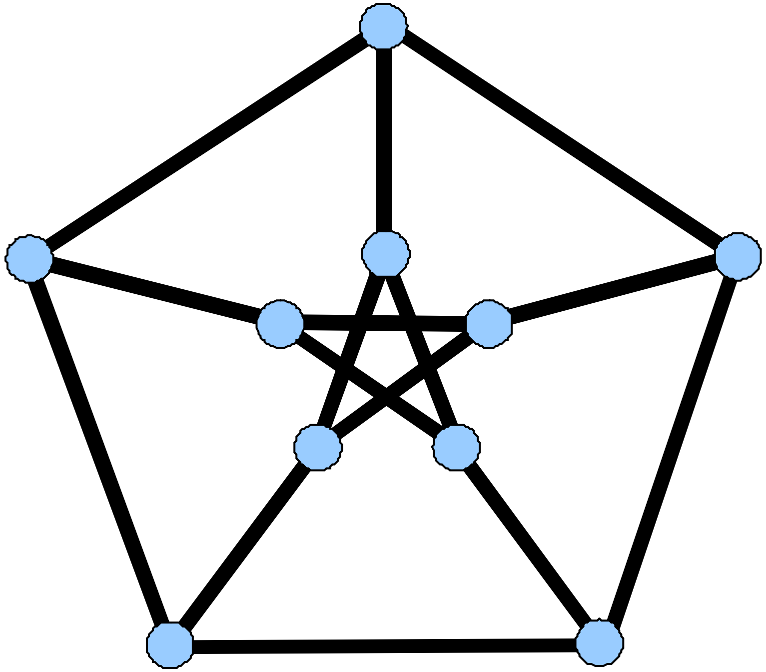


Why Sets or Matrices?

- A visual representation of a graph can be ambiguous; unless drawn carefully vertices can appear or disappear unintentionally.

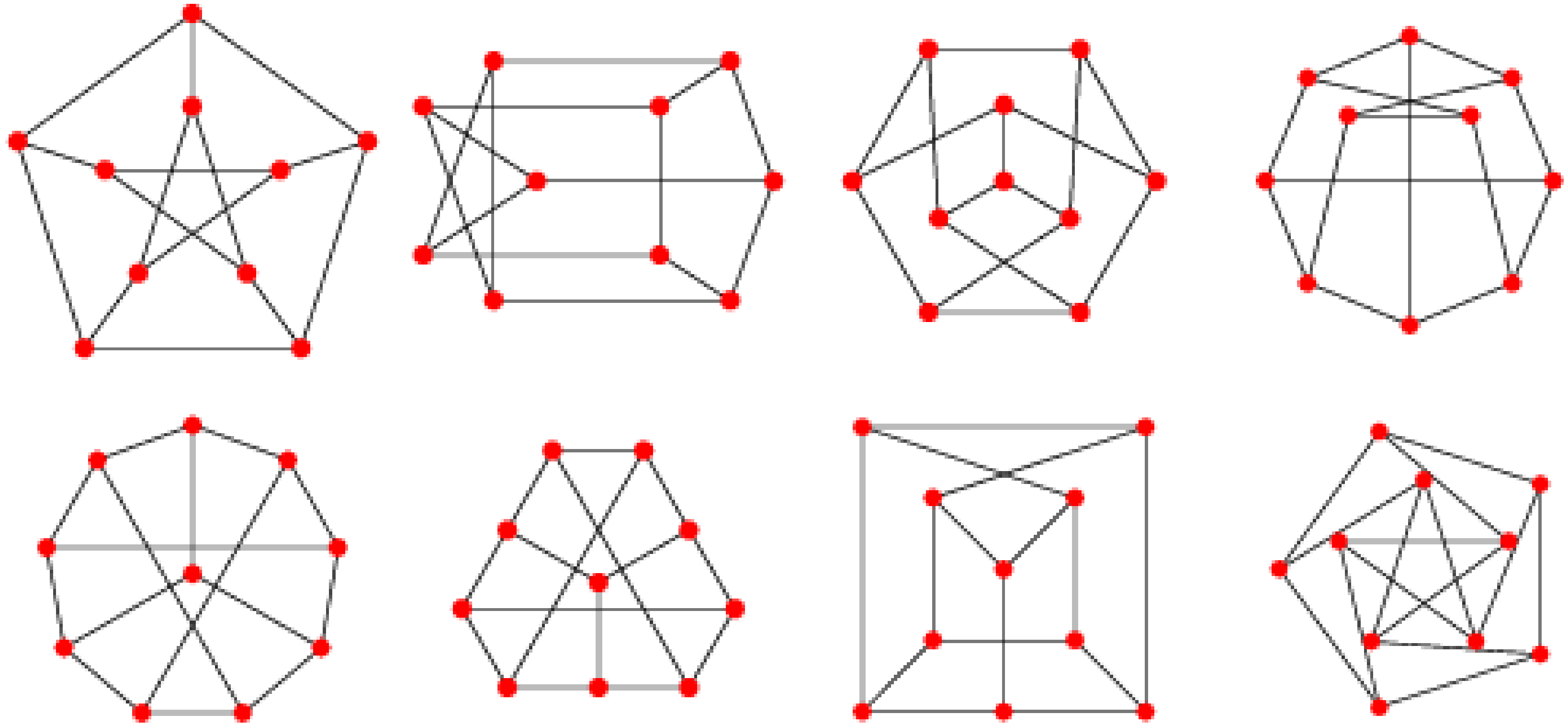


Why Visual?

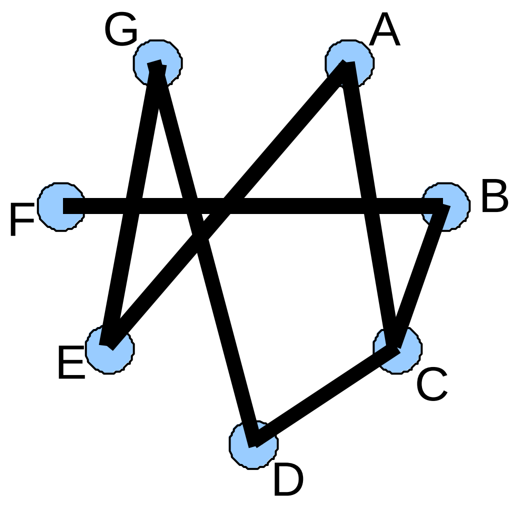


- We can quickly observe symmetry. A visual representation also disguises isomorphisms between graphs.

Petersen Graph representations



Matrix representations



- Recall the graph of Alan, Bob, Chad and their friends.
- The graph is crowded in places and hard to read so we use a matrix to describe it.

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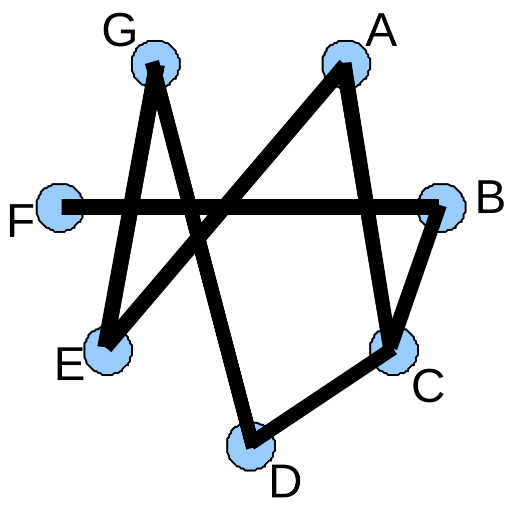
Matrix representations



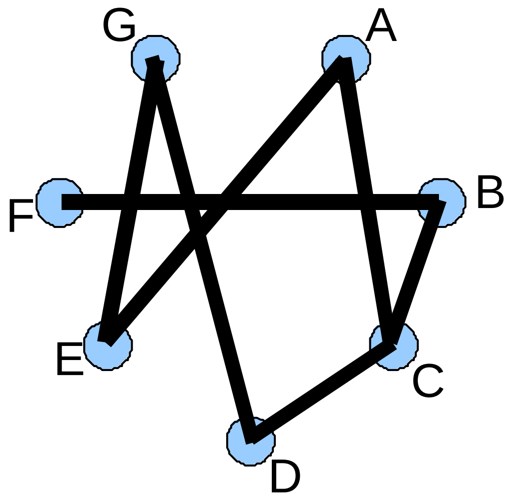
- The graph is defined by the relationships between the vertices.
- Each pair of vertices either does or does not have edge between them.

Definition

- Adjacency matrix – the number in cell (i,j) is the number of edges between vertex i and vertex j.

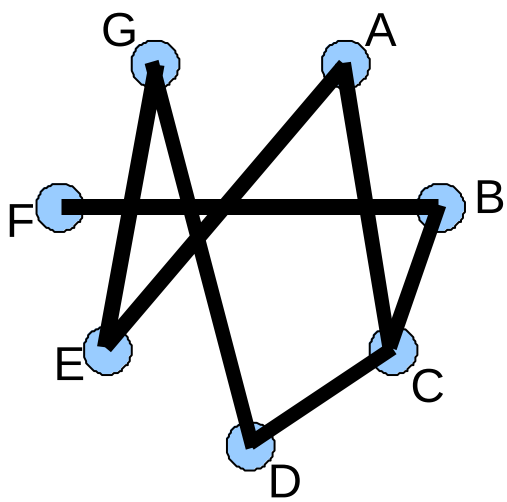
	A	B	C	D	E	F	G
A	0	0	1	0	1	0	0
B	0	0	1	0	0	1	0
C	1	1	0	1	0	0	0
D	0	0	1	0	0	0	1
E	1	0	0	0	0	0	1
F	0	1	0	0	0	0	0
G	0	0	0	1	1	0	0

Matrix representations



	A	B	C	D	E	F	G
A	0	0	1	0	1	0	0
B	0	0	1	0	0	1	0
C	1	1	0	1	0	0	0
D	0	0	1	0	0	0	1
E	1	0	0	0	0	0	1
F	0	1	0	0	0	0	0
G	0	0	0	1	1	0	0

Definition



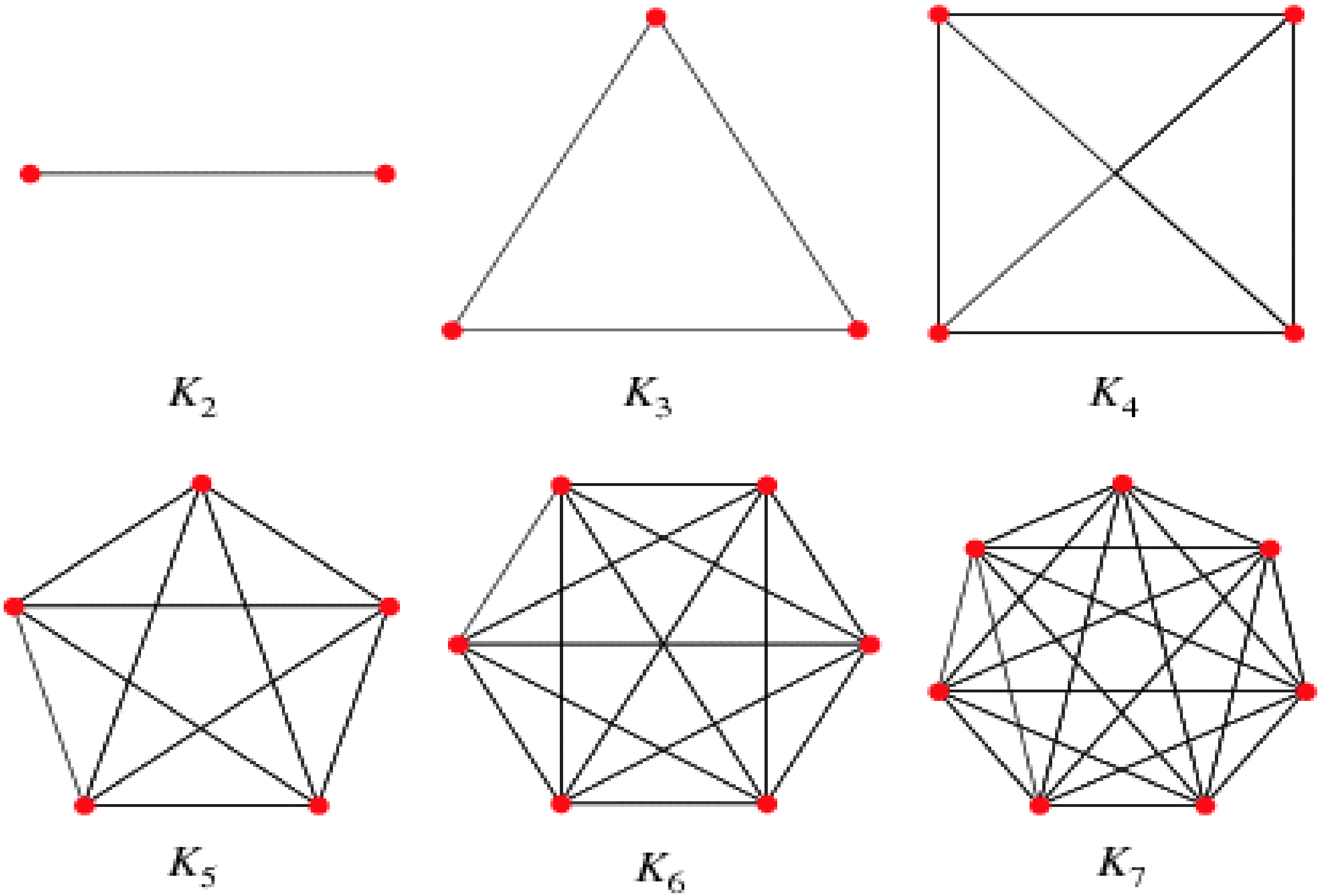
- Degree of a vertex is the "number of edges sticking out"
- Degree is the number of edges incident to a vertex
- Degree is the number of neighbors of a vertex

Definition

	A	B	C	D	E	F	G
A	0	0	1	0	1	0	0
B	0	0	1	0	0	1	0
C	1	1	0	1	0	0	0
D	0	0	1	0	0	0	1
E	1	0	0	0	0	0	1
F	0	1	0	0	0	0	0
G	0	0	0	1	1	0	0

- Degree of a vertex is the row sum (column sum)

Complete Graphs



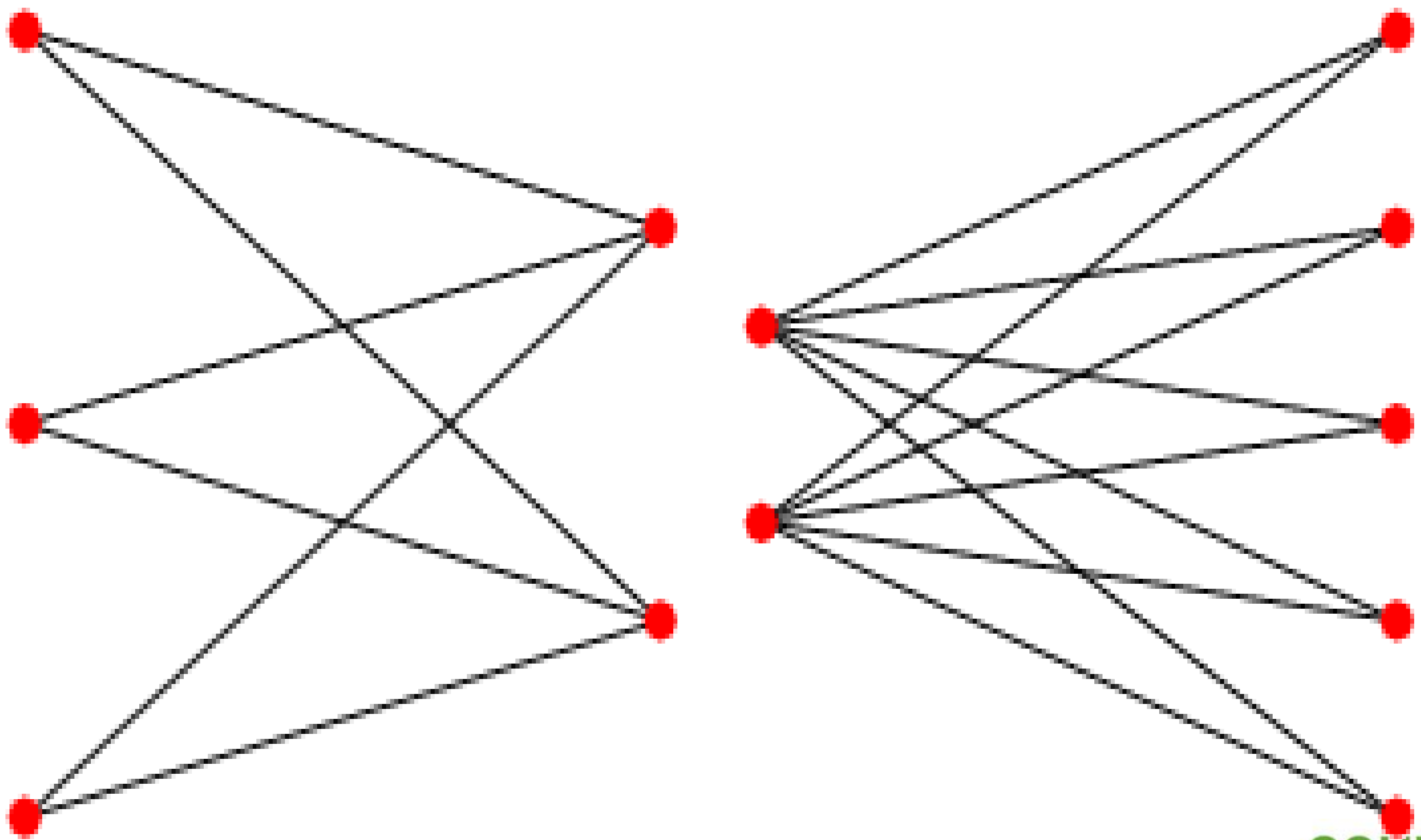
Complete Graphs

0	1
1	0

0	1	1
1	0	1
1	1	0

0	1	1	1
1	0	1	1
1	1	0	1
1	1	1	0

Complete Bipartite graphs

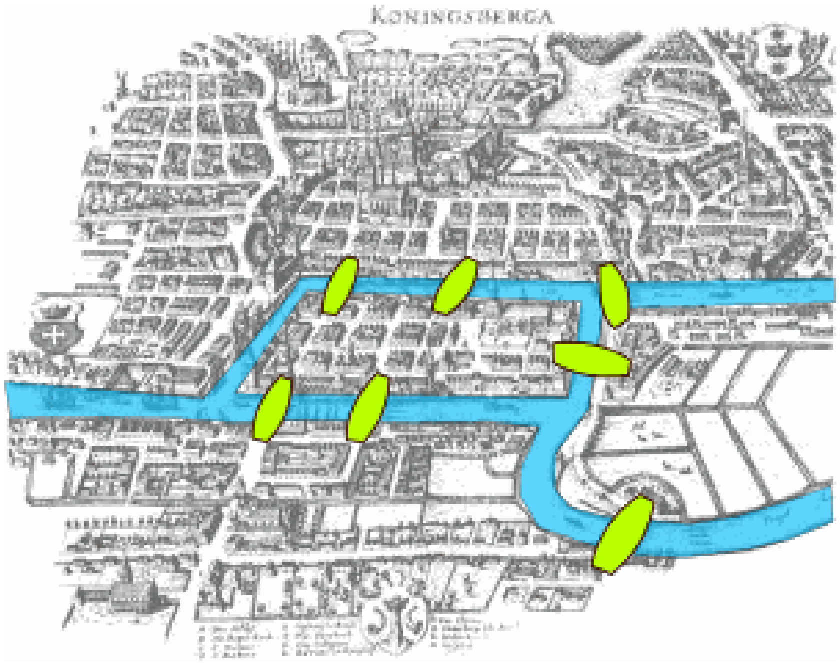


Complete Bipartite graphs

0	0	1	1	1
0	0	1	1	1
1	1	0	0	0
1	1	0	0	0
1	1	0	0	0

0	0	1	1	1	1	1
0	0	1	1	1	1	1
1	1	0	0	0	0	0
1	1	0	0	0	0	0
1	1	0	0	0	0	0
1	1	0	0	0	0	0
1	1	0	0	0	0	0

The Seven Bridges of Königsberg



- The city of Königsberg in Prussia was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges.

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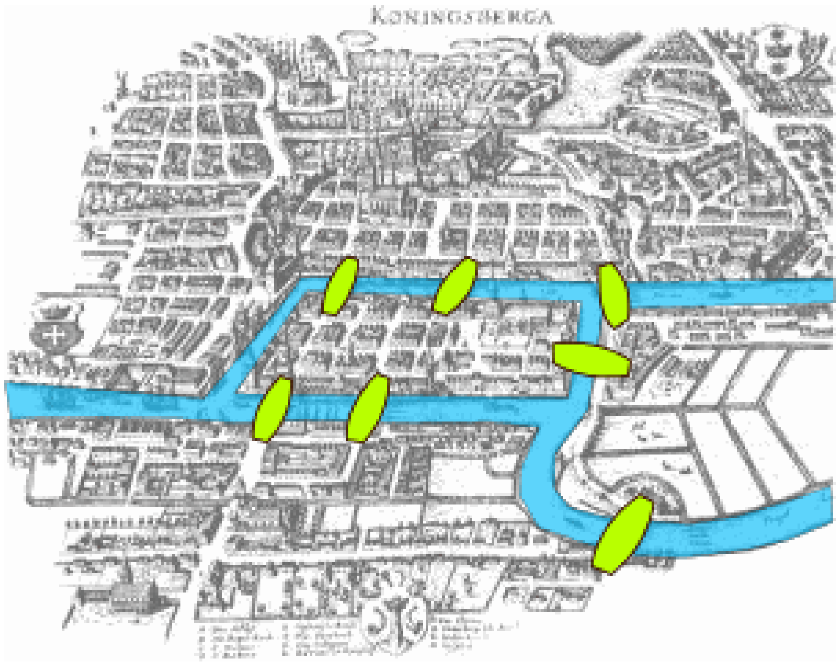
The Seven Bridges of Königsberg



- The problem was to find a walk through the city that would cross each bridge once and only once. The islands could not be reached by any route other than the bridges, and every bridge must have been crossed completely every time (one could not walk half way onto the bridge and then turn around and later cross the other half from the other side).

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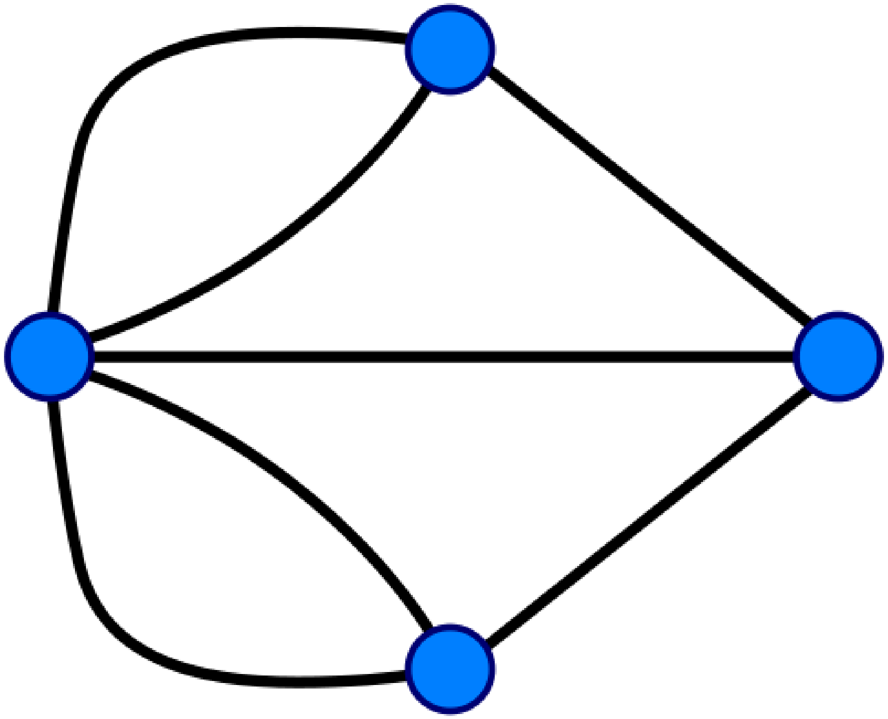
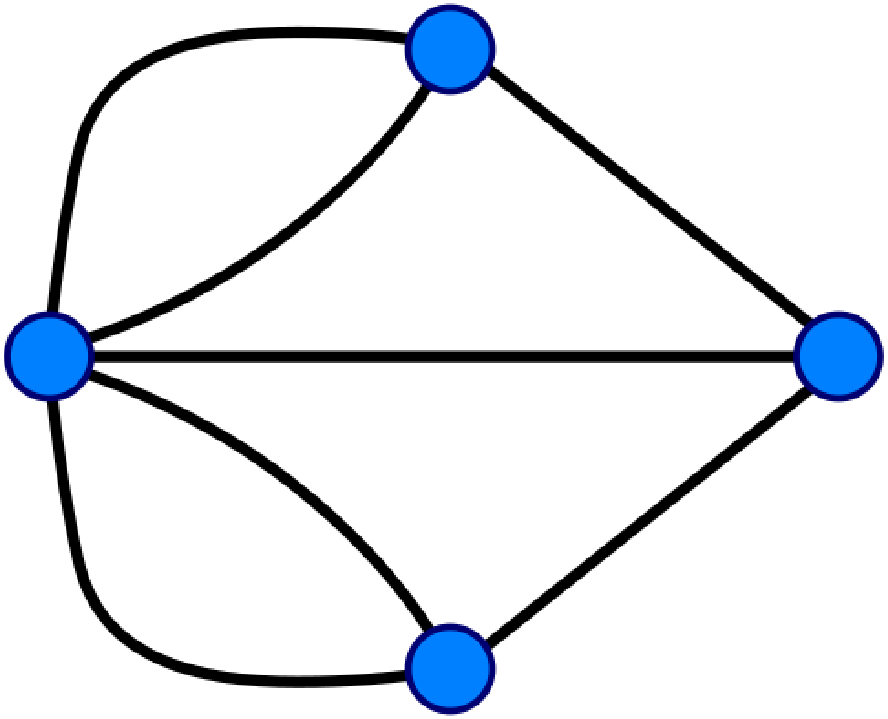
The Seven Bridges of Königsberg



- We represent the land masses as vertices of the graph and bridges as edges of the graph.

Euler circuit



- An Euler circuit in a graph is a sequence of vertices and incident edges that begin and end at the same vertex, and visits each edge EXACTLY once.

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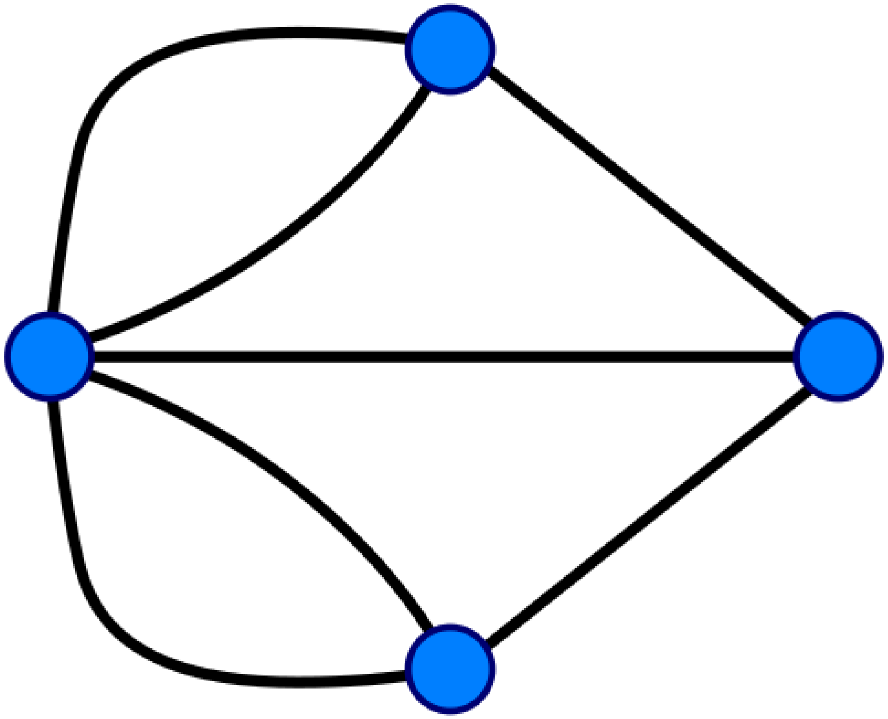
The Seven Bridges of Königsberg



- The problem was to find an Euler circuit in the graph.

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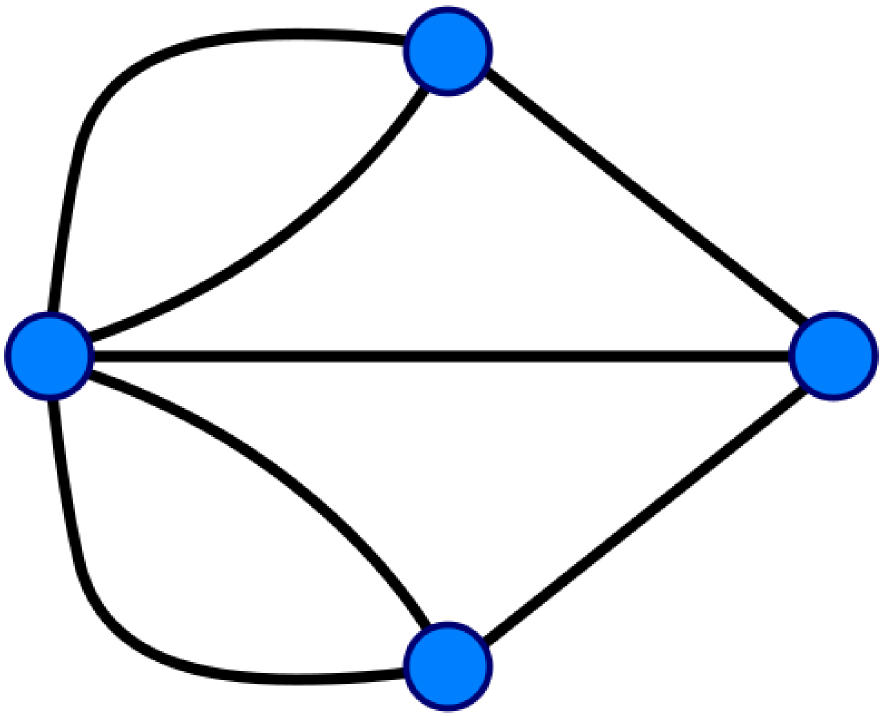
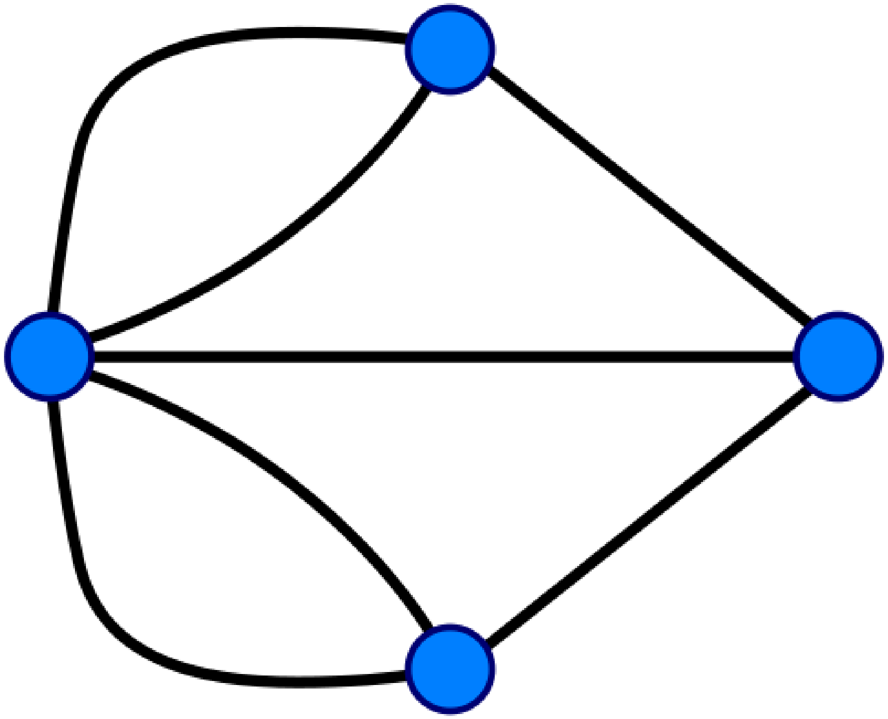
Euler circuits



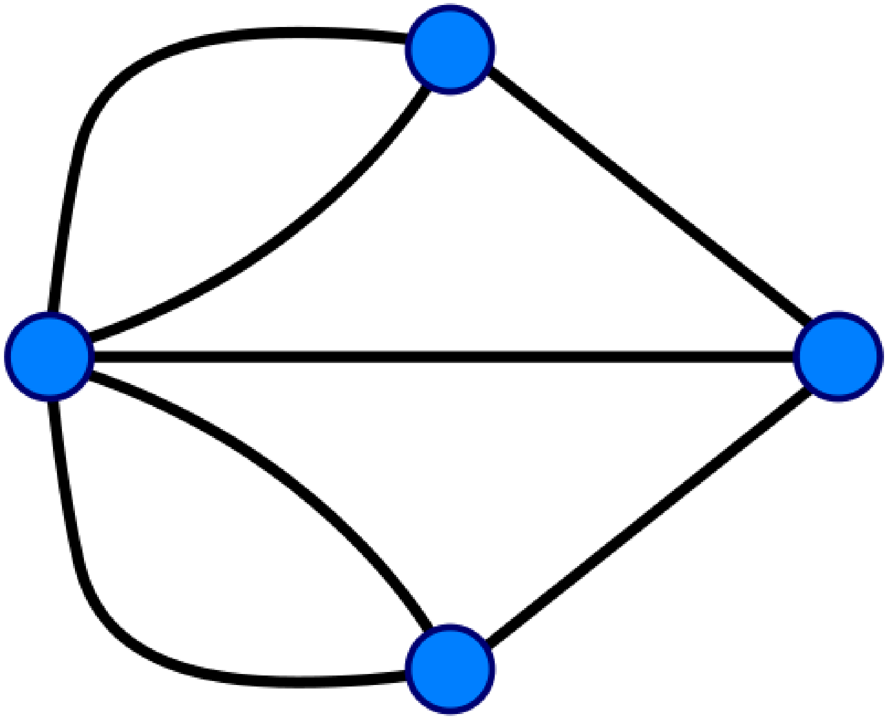
- In 1735, Euler showed that there was no solution to the seven bridges problem.

Euler Circuits



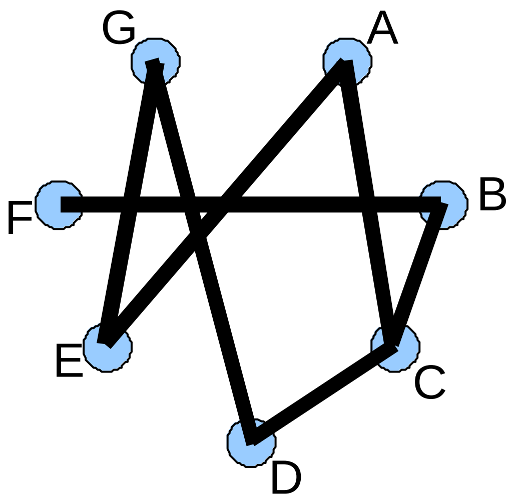
- Euler showed that for a graph to have an Euler circuit, all of the vertices have to have even degree (an even number of edges sticking out of each vertex). The graph also has to be connected

Euler circuits



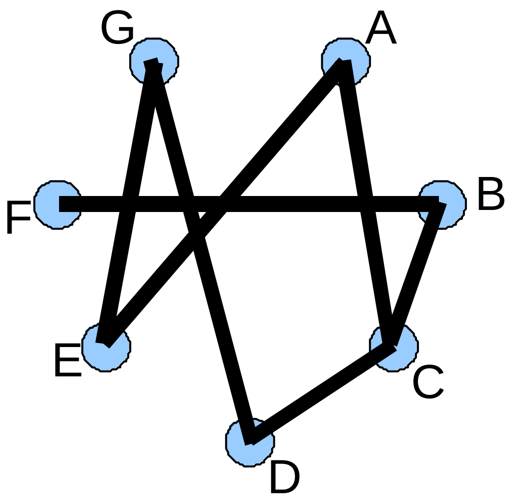
0	2	1	0
2	0	1	2
1	1	0	1
0	2	1	0

Matchings in graphs



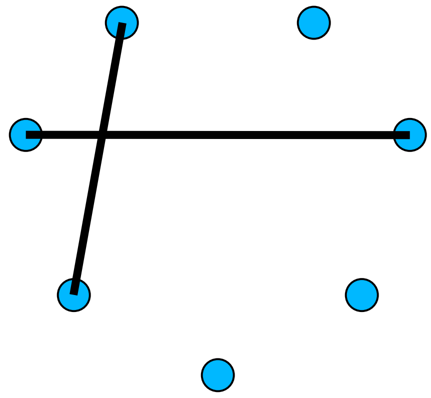
- Alan, Bob and the gang are taking an online interviewing class together, and because of geography, only certain members can meet each other.

Matchings in graphs



- Each interview will use a distinct pair of people from the class, and ideally, we'd like to schedule as many interviews as possible.

Matchings in graphs



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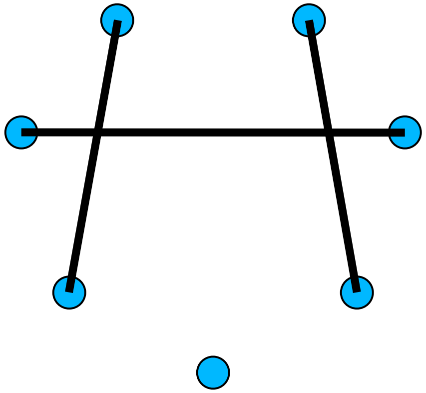

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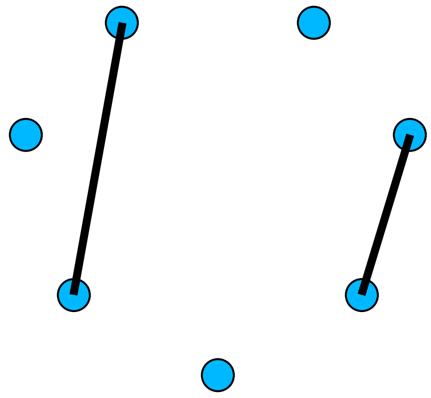

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Matchings in graphs

	A	B	C	D	E	F	G
A	0	0	1	0	1	0	0
B	0	0	1	0	0	1	0
C	0	0	0	1	0	0	0
D	0	0	0	0	0	0	1
E	0	0	0	0	0	0	1
F	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0

- Because each relationship is represented twice in the matrix, we’ll create an upper triangular matrix (eliminate the duplicates).

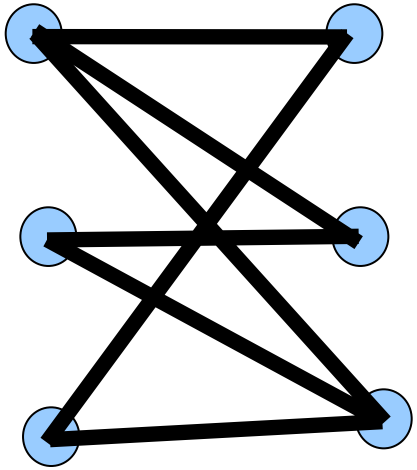
Matchings in graphs

	A	B	C	D	E	F	G
A	0	0	0	0	1	0	0
B	0	0	0	0	0	1	0
C	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0
E	0	0	0	0	0	0	1
F	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0

- Each matching will have at most one 1 in any row or column

Perfect matching

- If every vertex is used in a matching, it is called perfect

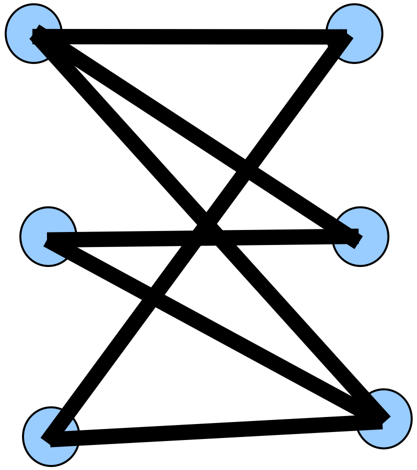


Perfect matching

0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Perfect matching

- Pair jobs and employees, given some set of restrictions.



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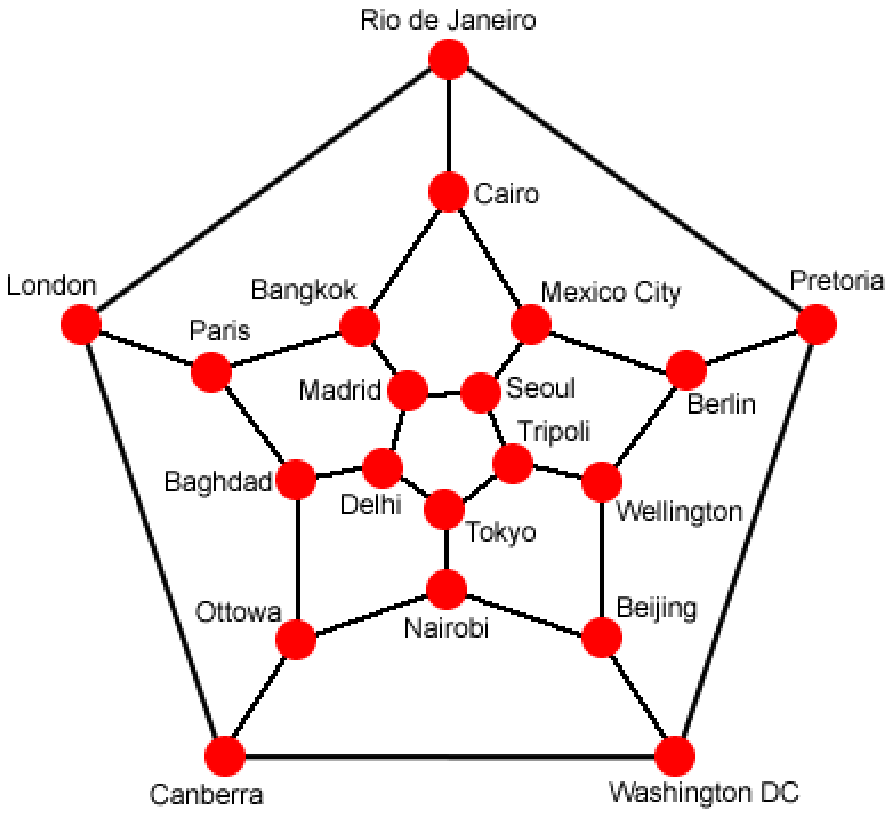
Hamilton's puzzle



- In 1859, Hamilton posed the problem of a traveler who wished to visit each of the twenty cities listed using only the indicated edges as travel routes.

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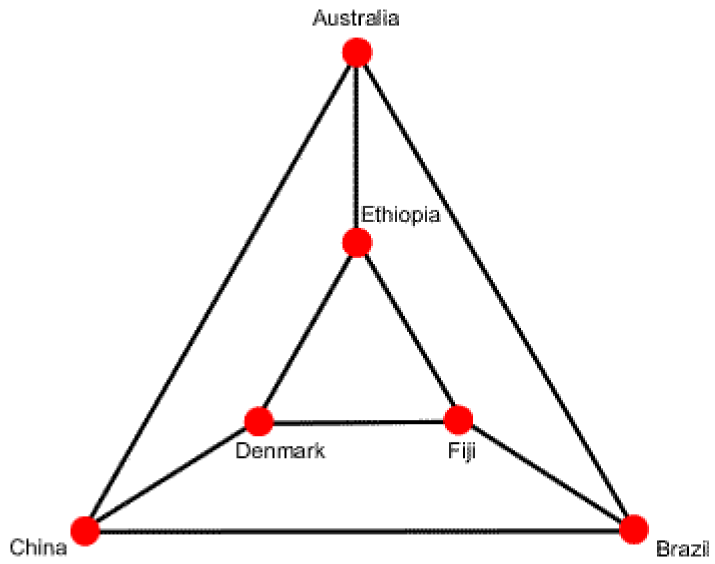
Traveling salesman problem



- In 1930, the problem of the traveling salesman is posed: he wishes to start at his home, visit all the points in his area and return home as quickly as possible.

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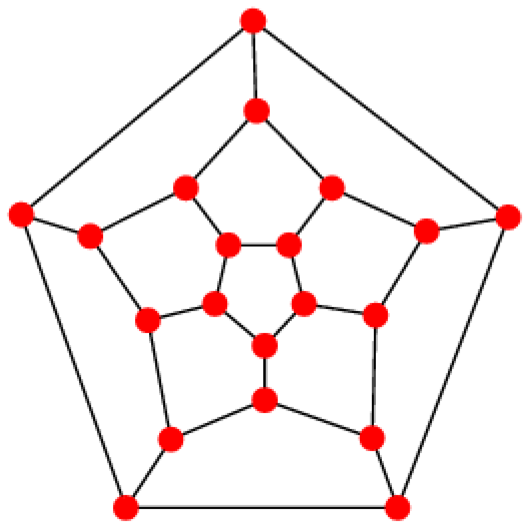
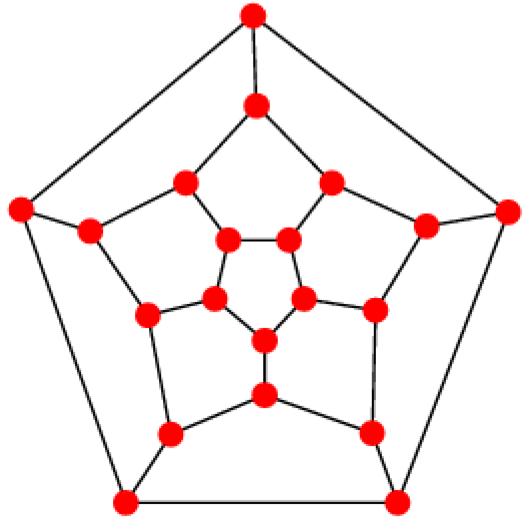
Hamilton cycle



- The TSP and Hamilton's puzzle are both asking us to find a Hamilton cycle in a graph. (The TSP will often vary the "lengths" of the edges as a challenge.)

Hamilton cycle

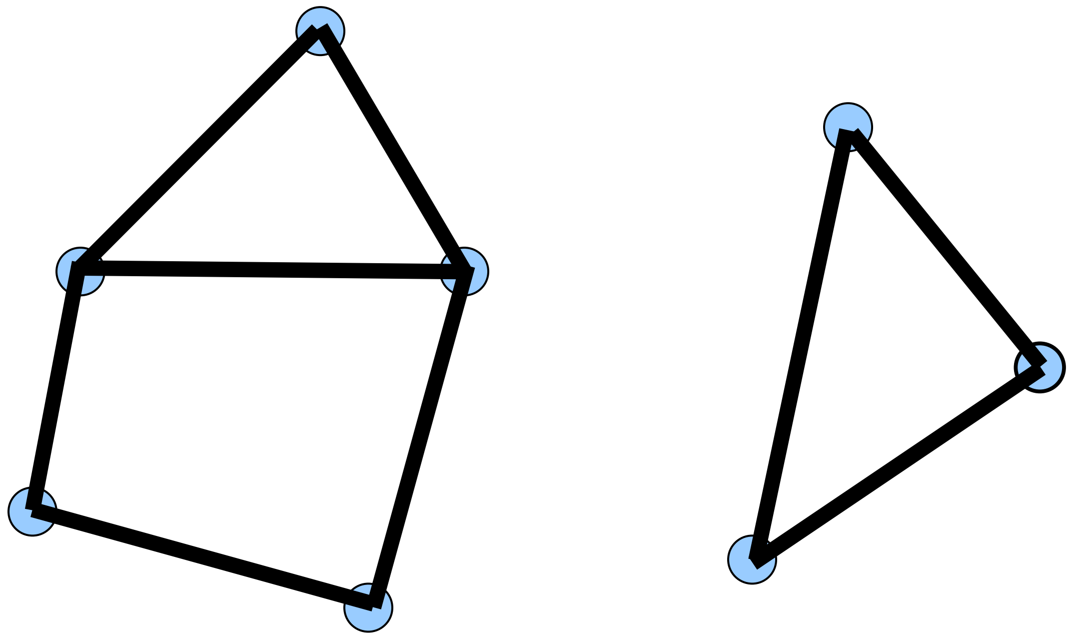


- A Hamilton cycle in a graph is a sequence of vertices and incident edges that visit each vertex EXACTLY once.

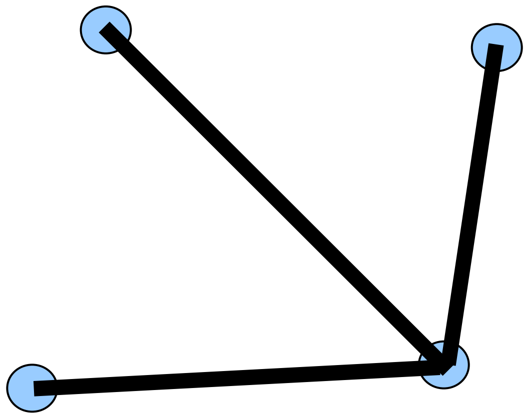
Hamilton cycle

- Determining whether or not a given graph has a Hamilton cycle is an NP-complete problem.
- We can determine some conditions for the presence of a Hamilton cycle.

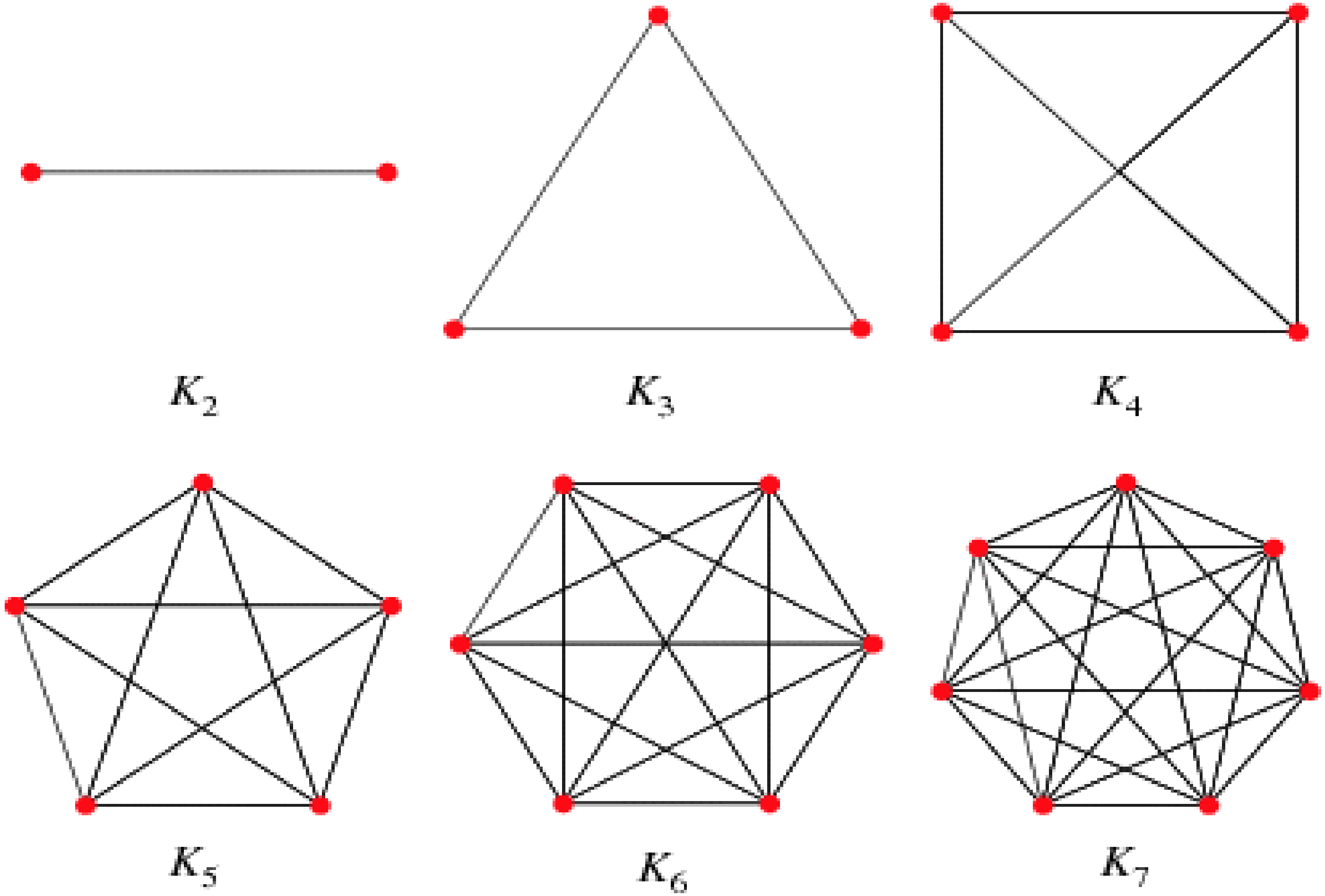
Hamilton cycle



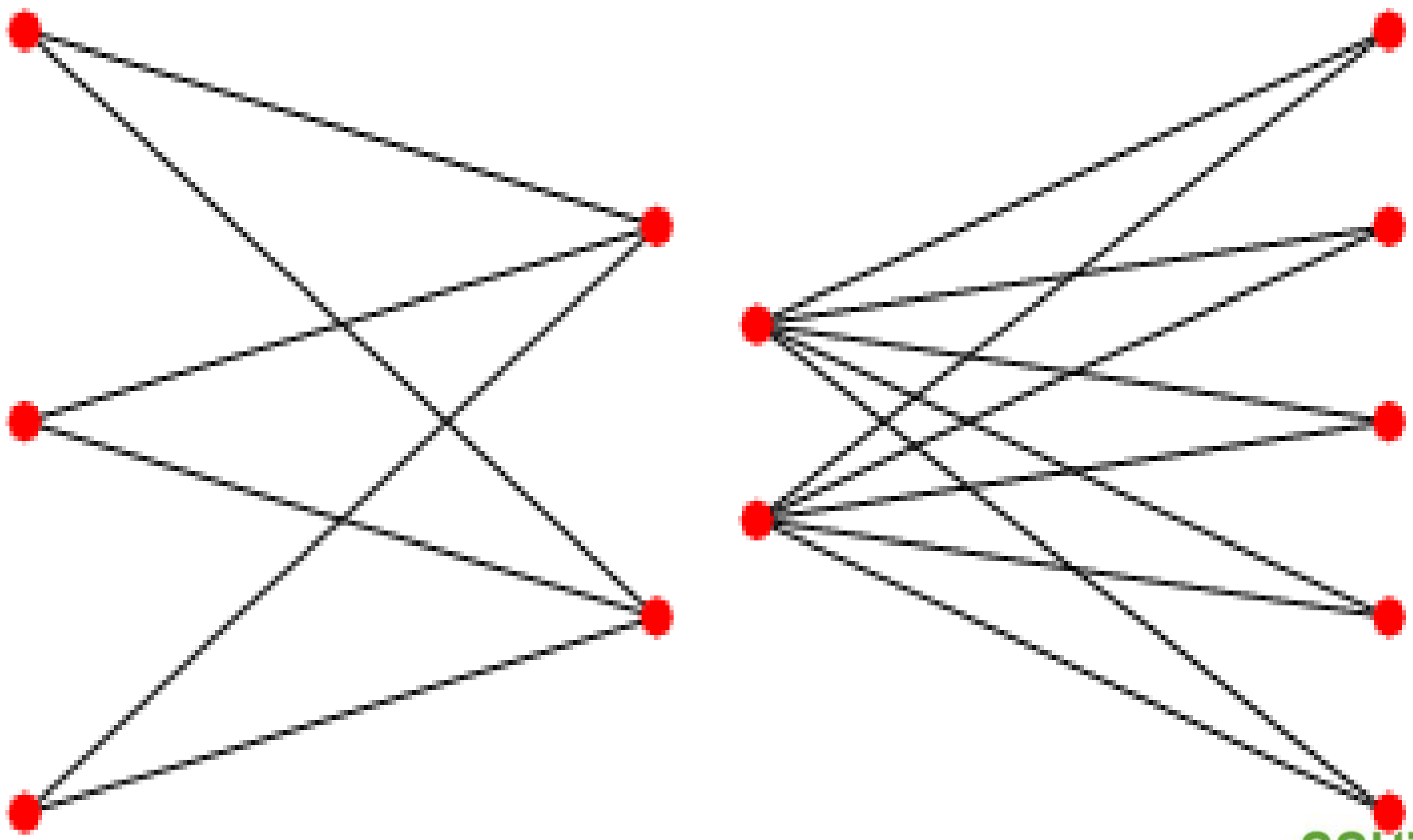
Hamilton cycle



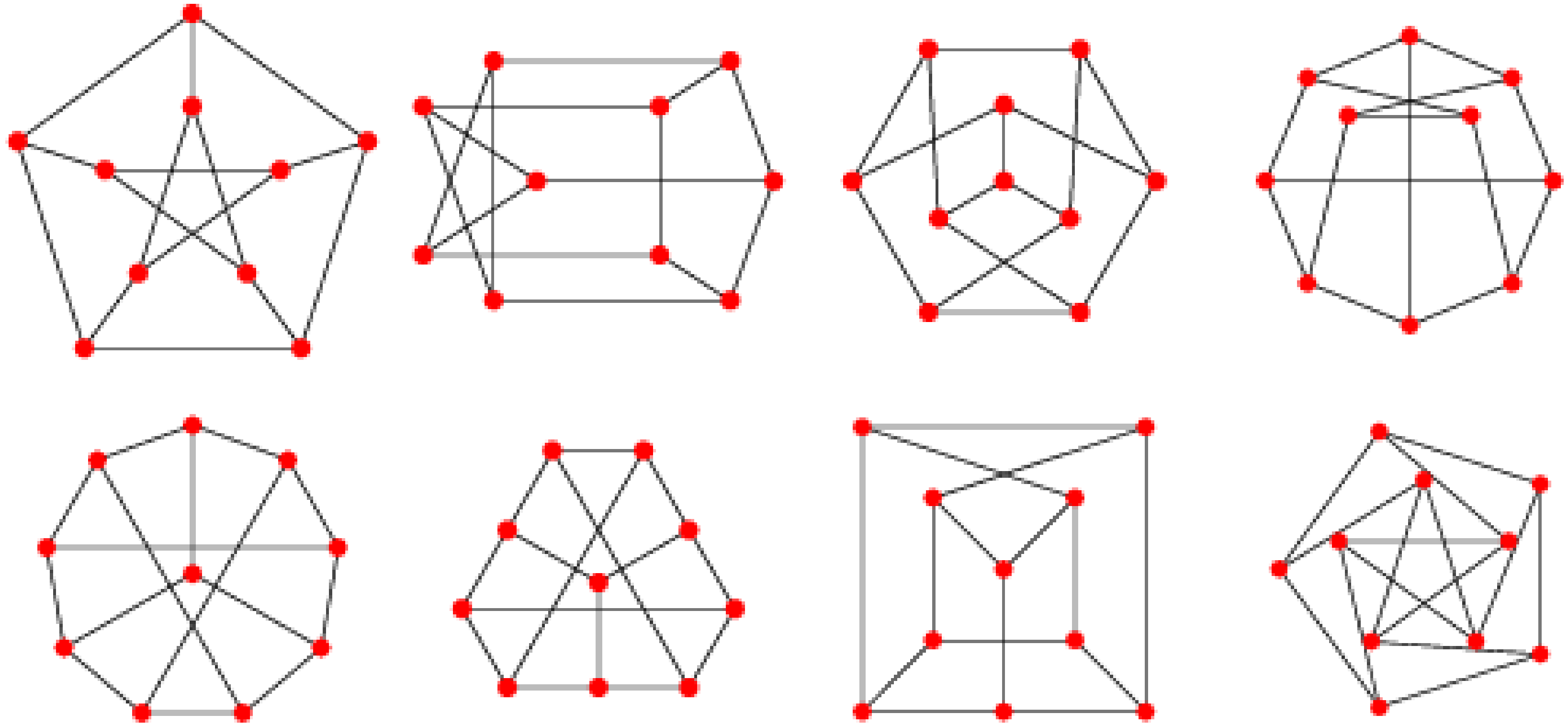
Complete Graphs



Complete Bipartite graphs



Petersen Graph representations



Example

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- E(G)={(Alan, Chad),(Dave, George), (Ed, Alan), (Bob, Fred), (Bob, Chad), (Chad, Dave), (George, Ed)}