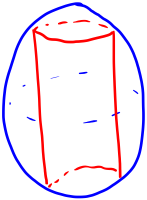
April 27 Math 1190 sec. 63 Spring 2017

Section 4.7: Optimization

Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius 10.



A cylinder has 2 dimensions, radius and height.

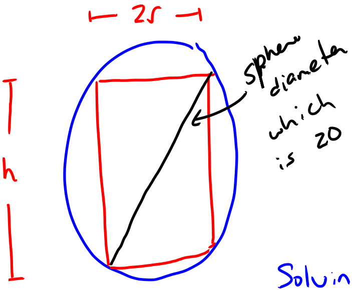
Call these r and h.

The volume $V = \pi r^2 h$

To get the largest volume, we want to maximize V.

Our constraint is that the cylinder has to fit in the sphere.

If we take a cross section of the sphere w/ the cylinder, we get



By the Pythagorean Thm

$$(2r)^2 + h^2 = 20^2$$

We reducing $V = \pi r^2 h$

Solving for r^2

$$4r^2 + h^2 = 20^2$$

$$4r^2 = 20^2 - h^2$$

$$r^2 = \frac{1}{4}\left(20^2 - h^2\right)$$

So $V = \pi r^2 h = \pi\left(\frac{1}{4}(20^2 - h^2)\right)h$

$$V = \frac{\pi}{4}(20^2 h - h^3)$$

Let's find critical number(s).

$$V'(h) = \frac{\pi}{4}(20^2 - 3h^2)$$

$V'(h)$ is always defined

$$V'(h) = 0 \Rightarrow \frac{\pi}{4}(20^2 - 3h^2) = 0$$

$$\Rightarrow 20^2 - 3h^2 = 0$$

$$\Rightarrow \quad 3h^2 = 20^2$$

$$h^2 = \frac{20^2}{3} \quad \Rightarrow \quad h = \frac{20}{\sqrt{3}} \quad \text{or} \quad \cancel{h = \frac{-20}{\sqrt{3}}}$$

$h > 0$ so ignore this one

We have one crit. # $h = \frac{20}{\sqrt{3}}$.

We'll use the 2nd der. test to see if V takes a max at this h.

$$V'(h) = \frac{\pi}{4}(20^2 - 3h^2)$$

$$V''(h) = \frac{\pi}{4}(-6h) = \frac{-6\pi}{4}h = \frac{-3\pi}{2}h$$

$$V''\left(\frac{20}{\sqrt{3}}\right) = \frac{-3\pi}{2} \cdot \frac{20}{\sqrt{3}} < 0 \quad \Rightarrow \text{ this } h \text{ maximizes } V.$$

$$V = \frac{\pi}{4}\left(20^2 h - h^3\right) \quad \text{so when} \quad h = \frac{20}{\sqrt{3}}$$

$$V\left(\frac{20}{\sqrt{3}}\right) = \frac{\pi}{4}\left(20^2 \cdot \frac{20}{\sqrt{3}} - \left(\frac{20}{\sqrt{3}}\right)^3\right)$$

$$= \frac{\pi}{4}\left(\frac{20^3}{\sqrt{3}} - \frac{20^3}{3\sqrt{3}}\right)$$

$$= \frac{\pi}{4}\left(\frac{20^3}{\sqrt{3}}\right)\left(1 - \frac{1}{3}\right)$$

$$= \frac{\pi}{4} \cdot \frac{20^3}{\sqrt{3}}\left(\frac{2}{3}\right) = \frac{20^3\pi}{6\sqrt{3}}$$

The cylinder of maximum Volume has volume $\frac{20^3\pi}{6\sqrt{3}}$ cubic units.

Random Questions

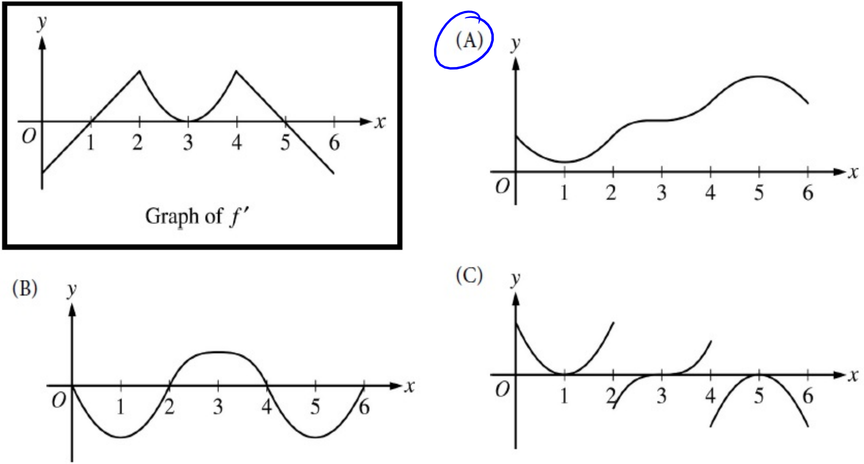


Figure: The graph of $f'(x)$ is shown in the upper left. Which of the three graphs could be the graph of $f(x)$?

1. $\lim_{x \to \pi} \frac{\cos x + \sin(2x) + 1}{x^2 - \pi^2}$ is

$\cos \pi + \sin(2\pi) + 1$

$-1 + 0 + 1 = 0$

" $\frac{0}{0}$ "

(A) $\frac{1}{2\pi}$

(B) $\frac{1}{\pi}$

(C) 1

(D) nonexistent

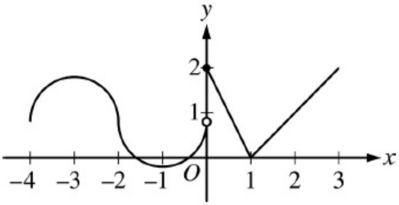
use l'H rule

$$\lim_{x \to \pi} \frac{-\sin x + \cos(2x) \cdot 2}{2x}$$

$$= \frac{-\sin \pi + 2\cos(2\pi)}{2\pi} = \frac{2}{2\pi}$$

$$= \frac{1}{\pi}$$

Figure: Evaluate the given limit.



Graph of f

. The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

If $f(x) = \int_1^{x^3} \frac{1}{1+\ln t}\,dt$ for $x \geq 1$, then $f'(2) =$

$$f'(x) = \frac{1}{1+\ln x^3} \cdot 3x^2$$

(a) $\dfrac{1}{1+\ln 2}$

(b) $\dfrac{12}{1+\ln 2}$

(c) $\dfrac{1}{1+\ln 8}$

(d) $\dfrac{12}{1+\ln 8}$

Find the derivative of the function

$$f(x) = \tan^2(\log_3 x).$$

(a) $f'(x) = \sec^4(\log_3 x)$

(b) $f'(x) = \dfrac{2\tan(\log_3 x)\sec^2(\log_3 x)}{x \ln 3}$

(c) $f'(x) = \dfrac{2}{x}\tan(\log_3 x)\sec^2(\log_3 x)$

(d) $f'(x) = \dfrac{\tan^3(\log_3 x)}{3}\dfrac{1}{x \ln 3}$