April 10 MATH 1112 sec. 54 Spring 2019

Section 7.5: Trigonometric Equations

In this section, we wish to consider **conditional** equations involving trigonometric functions. Our goal will be to find a solution set.

Some examples of trigonometric equations include

$$2\cos(x)-1=0, \qquad \sin\theta\cos\theta+\sin\theta=0, \qquad 2\tan^2 x-\tan x-1=0,$$

$$\csc 2\theta=\sec 2\theta, \qquad \tan^2(3x)=3, \quad \text{and so forth.}$$

We'll use trigonometric identities, our knowledge of some trig values, and inverse trigonometric functions as needed.

A Couple of Simple Examples

Find all possible solutions of the equation $2\cos(x) - 1 = 0$.

we'll start with some algebra. Isolate $\cos(x)$.

$$2\cos(x) - 1 = 0 \implies 2\cos(x) = 1 \implies$$

$$\cos(x) = \frac{1}{2}$$

There are infinitely many solutions because the cosine is periodic. Let's identify the solutions x in the interval $[0, 2\pi)$ - i.e. in one "rotation.".

$$\cos x = \frac{1}{2}$$

The cosine is positive in quadrants I and IV.

So there are two solutions in $[0, 2\pi)$.

In quadrant I we have $x = \frac{\pi}{3}$.

In quadrant IV we have $x = \frac{5\pi}{3}$.

If we add or subtract any integer multiple of 2π, the result is another solution.

So all solutions to $2\cos x - 1 = 0$

can be expressed as

$$x = \frac{\pi}{3} + 2\pi k$$

or

$$x = \frac{5\pi}{3} + 2\pi k$$

for $k = 0, \pm 1, \pm 2, \ldots$

$k = 0, \pm 1, \pm 2, \ldots$ can be written as $k \in \mathbb{Z}$

"k is in the integers"

Graphical Representation

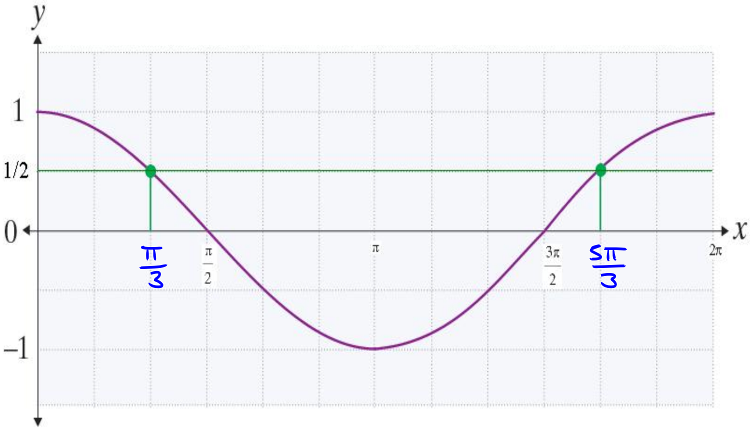


Figure: The solutions of $2\cos(x) - 1 = 0$ correspond to intersections of the curves $y = \cos x$ and $y = \frac{1}{2}$. Intersections continue to the left and right every 2π units.

Another Simple Example

Find all possible solutions of the equation $\sin(x) = \cos(x)$.

we can write the equation in terms of one trig function.

If $\cos(x) \neq 0$, we can divide by $\cos x$

$$\sin x = \cos x \quad \Rightarrow \quad \frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\tan x = 1$$

* If $\cos x = 0$, then $\sin x = 1$ or $\sin x = -1$

For x in $[0, 2\pi)$ there are 2 solutions in quadrants I and III.

The quadrant I solution is $x = \frac{\pi}{4}$.

The quadrant III solution is $x = \frac{5\pi}{4}$

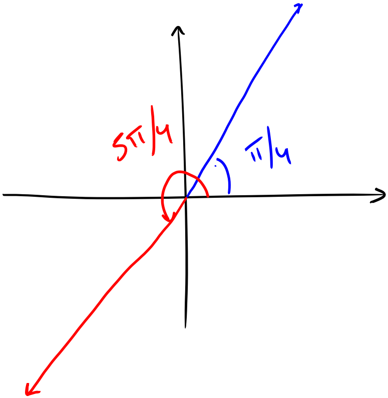
So all solutions are given by

$$x = \frac{\pi}{4} + 2\pi k \quad \text{or} \quad x = \frac{5\pi}{4} + 2\pi k$$

for $k = 0, \pm 1, \pm 2, \ldots$

Because the period of $\tan x$ is π, these can be combined as

$$x = \frac{\pi}{4} + k\pi \quad \text{for } k = 0, \pm 1, \pm 2, \ldots$$



Graphical Representation

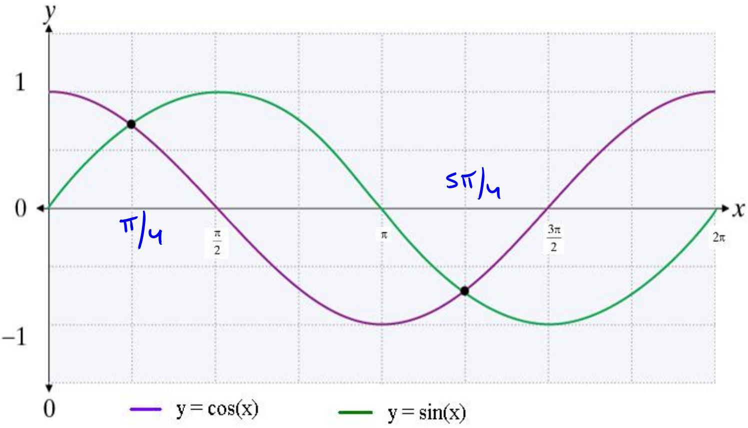


Figure: The solutions of $\sin(x) = \cos(x)$ correspond to intersections of the curves $y = \cos x$ and $y = \sin(x)$. Intersections continue to the left and right every 2π units.

A General Observation

When solving more complicated trigonometric equations, we will try to rewrite the problem in the form of **one or more** equations that look like

$$\text{One Trig Function} \quad = \quad \text{One Number}$$

We typically determine solution(s) in one period, and then extend those solutions if required.