Here is what review we did in class on Tuesday April 12.

The cubic spline problem was not completed, but the set up is here and the final solution is available

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(4) Consider using a 3^{rd} order interpolating polynomial $P_3(t)$ to approximate the function $f(t) = \tan^{-1} t$ on the interval $-1 \le t \le 1$. Find the *x*-values x_0, x_1, x_2, x_3 that will minimize the error.

(5) Use the fact that $|f^{(4)}(t)| \le 24$ for $-1 \le t \le 1$ to bound the error $|f(t) - P_3(t)|$ from problem (4).

4) The optimal nodes are the Chebyshev nodes;
for P3, the roots of T4.
$$X_j = Cos\left(\frac{(2j+1)\pi}{2\cdot4}\right) \quad j = 0, 1, 2, 3$$

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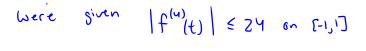
$$X_{b} = C_{0S} \left(\frac{\pi}{8}\right) = 0.9239$$

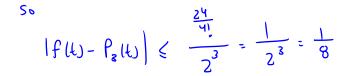
$$X_{1} = C_{0S} \left(\frac{3\pi}{8}\right) = 0.3827$$

$$X_{2} = C_{0S} \left(\frac{5\pi}{8}\right) = -0.3827$$

$$X_{3} = C_{0S} \left(\frac{4\pi}{8}\right) = -0.9239$$

5) Error
$$|f(t) - \frac{\beta_3}{3}(t)| \leq \frac{L}{2^3}$$
 where $L = \max\left|\frac{f^{(4)}}{4!}\right|$





= 0.125

< □ > < □ > < □ > < Ξ > < Ξ > < Ξ > Ξ のへで April 12, 2016 17 / 47 (6) Find the piece-wise linear interpolating function for the data set $\{(1,3), (1.5,2), (2,3.5)\}$.

(7) Find the natural cubic spline that interpolates the data in problem(6).

Line through (1,3) and (1.5,2) ()Slope $M = \frac{2 - 3}{1 - 5} = \frac{-1}{2 - 5} = -2$ y-3=-2(x-1) = -2x+2 = y= -2x+5 Line through (1.5,2) and (2,3.5) Slope $m = \frac{3.5-2}{2-1.5} = \frac{1.5}{0.5} = 3$ April 12, 2016 20/47

$$y - 2 = 3(x - 1.5) = 3x - 4.5$$

$$y = 3x - 2.5$$

$$s_{*} = \begin{cases} -2x + 5 \\ 3x - 2.5 \end{cases}, i \in x \in 1.5$$

$$(x)^{2} = \begin{cases} -2x + 5 \\ 3x - 2.5 \\ 1.5 \in x \in 2 \end{cases}$$

$$(1, 3), (1.5, 2) (2, 3.5)$$

$$(1, 3), (1.5, 2) (2, 3.5)$$

$$(1, 3), (1.5, 2) (2, 3.5)$$

$$(1, 3), (1.5, 2) (2, 3.5)$$

$$M_{1} + 4M_{2} + M_{3} = \frac{6}{h^{2}} \left(y_{3} - 2y_{2} + y_{1} \right)$$

$$= \frac{1}{9} \left(y_{3} - 2y_{2} + y_{1} \right)$$
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here h= 12

$$4 M_{2} = \frac{6}{(\frac{1}{2})^{2}} (3.5 - 2(2) + 3)$$
$$= \frac{6}{\frac{1}{4}} (6.5 - 4) = 24 (2.5)$$
$$M_{2} = \frac{24}{4} (2.5) = 6 \cdot (\frac{5}{2}) = 15$$

 $M_{1}=0$ $M_{2}=15$ $M_{3}=0$

 $x_{1} = 1$ $x_{2} = 1.5$ $x_{3} = 2$ $h = \frac{1}{2}$

 $y_1 = 3$ $y_2 = 2$ $y_3 = 3.5$

 $f_{0r} \int_{J^{2}}^{2} 1$ $O + \frac{15}{3} (x-1)^{3} + \frac{3}{1} \int_{J_{2}}^{J_{2}} (1.5 - x) + \frac{2}{1/2} (x-1) - \frac{1}{12} \int_{0}^{J_{2}} [0 + 15(x-1)]$ $S (x^{3} - 3x^{2} + 3x - 1) + 6(1.5 - x) + 4(x-1) - \frac{1}{12} [15(x-1)]$

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1= 2 $\frac{15}{2} (z - x)^{3} + \frac{2}{\frac{1}{12}} (z - x) + \frac{3.5}{\frac{1}{12}} (x - 1.5)$ $-\frac{1/2}{6}$ [15(2-x) +0] Expand and simplify completely

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(11) Use Gaussian numerical integration $I_2(f)$ to approximate

$$\int_{-1}^{1} \sqrt[3]{x} e^{-x} dx.$$

 $\overline{L}_{1}(f) = w_{1} f(x_{1}) + w_{2} f(x_{2})$

$$f(x) = \sqrt[3]{x} e^{-x}$$

$$f(x_{1}) = \sqrt[3]{\frac{-1}{\sqrt{3}}} e^{-(\frac{1}{\sqrt{3}})}$$

$$f(x_{1}) = \sqrt[3]{\frac{-1}{\sqrt{3}}} e^{-(\frac{1}{\sqrt{3}})}$$

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$$f(x_2) = 3 \frac{1}{\sqrt{3}} e^{-\left(\frac{1}{\sqrt{3}}\right)} = \frac{1}{\sqrt{3}} e^{\frac{1}{\sqrt{3}}} e^{\frac{1}{\sqrt{3}}}$$

 $T_{2}(f) = \frac{-1}{6\sqrt{3}}e^{\frac{1}{13}} + \frac{1}{6\sqrt{3}}e^{\frac{1}{13}}$

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