Here is what review we did in class on Tuesday April 12.

The cubic spline problem was not completed, but the set up is here and the final solution is available

HERE <--this is a link

(4) Consider using a 3^{rd} order interpolating polynomial $P_3(t)$ to approximate the function $f(t) = \tan^{-1} t$ on the interval $-1 \le t \le 1$. Find the x-values x_0, x_1, x_2, x_3 that will minimize the error.

(5) Use the fact that $|f^{(4)}(t)| \le 24$ for $-1 \le t \le 1$ to bound the error $|f(t) - P_3(t)|$ from problem (4).

4) The optimal nodes are the Chebyshev nodes; for P_3, the roots of T_4.

$$x_j = \cos\left(\frac{(2j+1)\pi}{2 \cdot 4}\right) \qquad j = 0, 1, 2, 3$$

$$x_0 = \cos\left(\frac{\pi}{8}\right) \doteq 0.9239$$

$$x_1 = \cos\left(\frac{3\pi}{8}\right) \doteq 0.3827$$

$$x_2 = \cos\left(\frac{5\pi}{8}\right) \doteq -0.3827$$

$$x_3 = \cos\left(\frac{7\pi}{8}\right) \doteq -0.9239$$

5) Error $\left| f(t) - P_3(t) \right| \leq \dfrac{L}{2^3}$ where

$$L = \max \left| \frac{f^{(4)}(t)}{4!} \right|$$

We're given $\left|f^{(4)}(t)\right| \leq 24$ on $[-1,1]$

So

$$\left|f(t) - P_3(t)\right| \leq \frac{\frac{24}{4!}}{2^3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\doteq 0.125$$

(6) Find the piece-wise linear interpolating function for the data set $\{(1,3),(1.5,2),(2,3.5)\}$.

(7) Find the natural cubic spline that interpolates the data in problem (6).

(6) Line through $(1,3)$ and $(1.5,2)$

Slope $m = \frac{2-3}{1.5-1} = \frac{-1}{0.5} = -2$

$y - 3 = -2(x-1) = -2x+2 \Rightarrow y = -2x+5$

Line through $(1.5,2)$ and $(2,3.5)$

Slope $m = \frac{3.5-2}{2-1.5} = \frac{1.5}{0.5} = 3$

$$y-2=3(x-1.5)=3x-4.5$$

$$y=3x-2.5$$

So $$\ell(x)=\begin{cases}-2x+5, & 1\le x\le 1.5\\ 3x-2.5, & 1.5\le x\le 2\end{cases}$$

7) $(1,3)$, $(1.5,2)$ $(2,3.5)$

We need M_1, M_2, M_3 but $M_1=M_3=0$

$$\underset{0}{\underset{\shortparallel}{M_1}}+4M_2+\underset{0}{\underset{\shortparallel}{M_3}}=\frac{6}{h^2}(y_3-2y_2+y_1)$$

here $h = \frac{1}{2}$

$$4M_2 = \frac{6}{\left(\frac{1}{2}\right)^2}\left(3.5 - 2(2) + 3\right)$$

$$= \frac{6}{1/4}\left(6.5 - 4\right) = 24\,(2.5)$$

$$M_2 = \frac{24}{4}(2.5) = 6 \cdot \left(\frac{5}{2}\right) = 15$$

$M_1 = 0 \qquad M_2 = 15 \qquad M_3 = 0$

$x_1 = 1 \qquad x_2 = 1.5 \qquad x_3 = 2 \qquad\qquad h = \frac{1}{2}$

$y_1 = 3 \qquad y_2 = 2 \qquad y_3 = 3.5$

for $j = 1$

$$0 + \frac{15}{3}(x-1)^3 + \frac{3}{1/2}(1.5 - x) + \frac{2}{1/2}(x-1) - \frac{1/2}{6}\left[0 + 15(x-1)\right]$$

$$5\left(x^3 - 3x^2 + 3x - 1\right) + 6(1.5 - x) + 4(x-1) - \frac{1}{12}\left[15(x-1)\right]$$

$j=2$

$$\frac{15}{3}(2-x)^3+\frac{2}{1/2}(2-x)+\frac{3.5}{1/2}(x-1.5)$$

$$-\frac{1/2}{6}\left[15(2-x)+0\right]$$

Expand and simplify completely

(11) Use Gaussian numerical integration $I_2(f)$ to approximate

$$\int_{-1}^{1} \sqrt[3]{x}\, e^{-x}\, dx.$$

$$I_2(f) = w_1 f(x_1) + w_2 f(x_2)$$

$$f(x) = \sqrt[3]{x}\; e^{-x}$$

$$f(x_1) = \sqrt[3]{\frac{-1}{\sqrt{3}}}\; e^{-\left(\frac{-1}{\sqrt{3}}\right)} = \frac{-1}{\sqrt[6]{3}}\; e^{\frac{1}{\sqrt{3}}}$$

$$f(x_2) = \sqrt[3]{\frac{1}{\sqrt{3}}}\; e^{-\left(\frac{1}{\sqrt{3}}\right)} = \frac{1}{\sqrt[6]{3}}\, e^{-\frac{1}{\sqrt{3}}}$$

$$I_2(f) = \frac{-1}{\sqrt[6]{3}}\, e^{\frac{1}{\sqrt{3}}} + \frac{1}{\sqrt[6]{3}}\, e^{-\frac{1}{\sqrt{3}}}$$

$$\doteq -1.0158$$