April 17 MATH 1112 sec. 54 Spring 2019

Section 8.1: The Laws of Sines and Cosines

An **oblique** triangle is one that does not have a right angle in it.
By "**solve** an oblique triangle," we mean finding the measure of each of its three sides and each of its three angles.

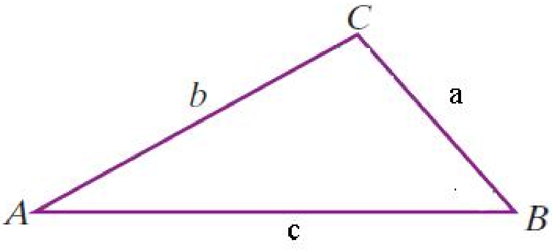


Figure: We will use the labeling convention that the angles are A, B, and C, and the sides opposite are labeled with the corresponding lower case a, b, and c.

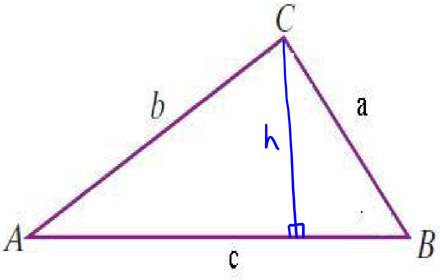
Solving an Oblique Triangle

We **must** have three pieces of information (sides/angles). And **at least one** piece of information MUST be a side length. There are four possibilities:

- Two angles + one side (AAS or ASA),
- Two sides and a non-included angle (SSA),
- Two sides and the angle between them (SAS), or
- Three sides (SSS).

The Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Let's establish that $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ using the diagram.



Drop an altitude from C of length h

From the right triangles

$\sin A = \dfrac{h}{b}$ and

$\sin B = \dfrac{h}{a}$

The Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Solving for h

$h = b\sin A$ and $h = a\sin B$

Since $h=h$, $b\sin A = a\sin B$

Divide both sides by ab

$$\frac{b\sin A}{ab} = \frac{a\sin B}{ab}$$ cancel like factors

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

This is readily extended to show $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

The Law of Sines

In order to use the Law of Sines, we must know one angle-side pair (e.g. A and a). Since each angle is greater than 0° and less than 180°, all sine values are positive. So the law can be stated as

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

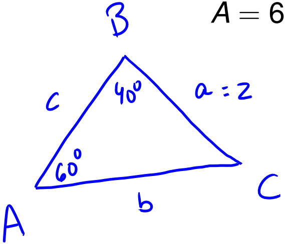
or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example (AAS)

Solve the triangle with the given information

$$A = 60^\circ, \quad B = 40^\circ, \quad a = 2$$



Find C: Since $A + B + C = 180^\circ$

$$C = 180^\circ - A - B = 180^\circ - 60^\circ - 40^\circ = 80^\circ$$

Find b: By the Law of sines

$$\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow b = \frac{a}{\sin A} \sin B$$

$$b = \frac{2}{\sin(60^\circ)} \sin(40^\circ) \approx 1.48$$

Since b is approximate and a is exact, let's use a to find c.

Find c: By the Law of Sines $\frac{c}{\sin C} = \frac{a}{\sin A}$

$$c = \frac{a}{\sin A}\sin C = \frac{2}{\sin 60^\circ}\sin 80^\circ \approx 2.27$$

The sides and angles are

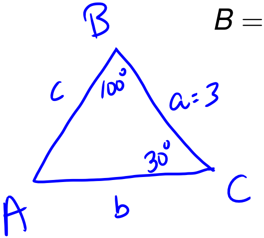
$a = 2$, $b = 1.48$, $c = 2.27$

$A = 60^\circ$, $B = 40^\circ$, $C = 80^\circ$

Example (ASA)

Solve the triangle with the given information

$$B = 100^\circ, \quad C = 30^\circ, \quad a = 3$$



We have to find A first to know a side-angle pair.

Find A : $A = 180^\circ - B - C = 180^\circ - 100^\circ - 30^\circ = 50^\circ$

Find b: $\dfrac{b}{\sin B} = \dfrac{a}{\sin A} \Rightarrow b = \dfrac{a}{\sin A} \sin B$

$$b = \frac{3}{\sin 50^\circ} \sin 100^\circ \approx 3.86$$

Find c: $\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow c = \frac{a}{\sin A} \sin C$

$$c = \frac{3}{\sin 50^\circ} \sin 30^\circ \approx 1.96$$

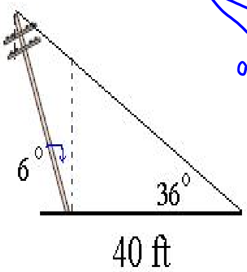
All sides and angles are

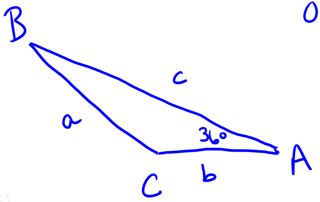
$a = 3$, $b = 3.86$, $c = 1.96$

$A = 50^\circ$, $B = 100^\circ$, $C = 30^\circ$

Application

A telephone pole was hit by a car and now leans 6° from the vertical. A point 40 ft from the base has an angle of elevation of 36° to the top of the pole. How tall is the pole?





Our question is what is a?

Given $C = 96°$ and $b = 40\text{ ft}$

we can use $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ if we find B

$$B = 180^\circ - A - C = 180^\circ - 36^\circ - 96^\circ = 48^\circ$$

Given $b = 40\text{ft}$

$$\frac{a}{\sin 36^\circ} = \frac{40\text{ft}}{\sin 48^\circ}$$

$$\Rightarrow \quad a = \frac{40\text{ft}}{\sin 48^\circ} \sin 36^\circ \approx 31.6 \text{ ft}$$