April 26 MATH 1112 sec. 54 Spring 2019

Section 8.1: The Laws of Sines and Cosines

In order to use the Law of Sines, we must know one angle-side pair (e.g. A and a). Since each angle is greater than 0° and less than 180°, all sine values are positive. So the law can be stated as

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Sines can be used for AAS, ASA, or SSA.

Section 8.2: The Law of Cosines

Theorem: For the triangle labeled using the previous convention, all three of the following equations hold

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}$$

This can be used if two sides and an included angle (SAS) are known, or if the three sides (SSS) are known.

The Law of Cosines & The Pythagorean Theorem

Apply the law of cosines to a right triangle for which $C = 90^\circ$; see what it produces.

For hypotenuse c, and sides a, b

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = a^2 + b^2 - 2ab\cos(90^\circ)$$

$$= a^2 + b^2 - 2ab(0)$$

$$c^2 = a^2 + b^2$$

So the Pythagorean Thm is a special case of the Law of cosines.

Example (SSS)

Solve the triangle given $a = 5, \quad b = 2, \quad c = 6$

By the Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$5^2 = 2^2 + 6^2 - 2(2)(6)\cos A$$

$$25 = 40 - 24\cos A$$

$$-15 = -24\cos A \quad \Rightarrow \quad \cos A = \frac{15}{24} = \frac{5}{8}$$

$$A \approx 51.3^\circ$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$6^2 = 2^2 + 5^2 - 2(2)(5)\cos C$$

$$36 = 29 - 20\cos C$$

$$7 = -20\cos C \Rightarrow \cos C = \frac{-7}{20}$$

$$C \approx 110.5^\circ$$

$$B = 180^\circ - A - C = 180^\circ - 51.3^\circ - 110.5^\circ$$
$$= 18.2^\circ$$

$A = 51.3^\circ$ $B = 18.2^\circ$ $C = 110.5^\circ$

$a = 5$ $b = 2$ $c = 6$

Area of a Triangle

Theorem: The area of a triangle is one half the product of any two sides times the sine of the included angle. That is

$$\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A.$$

There is an alternative theorem that can be used if no angles are known.

Theorem: (Heron's Formula) For the triangle with sides of lengths a, b, and c. Define the semi-perimeter

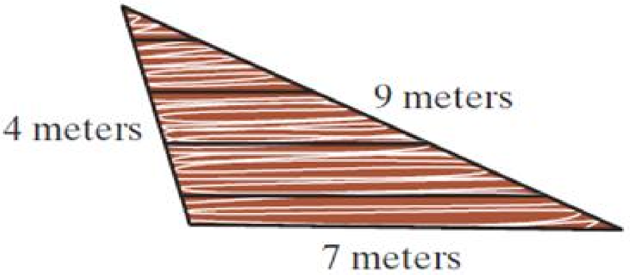
$$s = \frac{a+b+c}{2}.$$

The area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}.$$

Example

A set designer needs to estimate the amount of paint required to paint a triangular piece of wooden backdrop. Determine the area of the wood piece shown to the nearest tenth of a meter.



Use Heron's formula with $a=4$, $b=7$, $c=9$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+7+9) = 10$$

$$s-a = 10-4 = 6$$

$$s-b = 10-7 = 3$$

$$s-c = 10-9 = 1$$

$$(\text{Area})^2 = s(s-a)(s-b)(s-c) = 10(6)(3)(1) = 180$$

So the area

$$A = \sqrt{180} = 6\sqrt{5} \approx 13.4$$

The area is approximately $13.4\ m^2$