

## Section 7.3: Verifying Identities

The following equation states an Identity. I will *verify* that it is true.

$$\csc(x) - \sin(x) = \cos(x) \cot(x)$$

We'll apply identities to the left side to rewrite it until we end up with the right side.

$$\csc x - \sin x = \frac{1}{\sin x} - \sin x$$

$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

Common denominator

$$= \frac{1 - \sin^2 x}{\sin x}$$

using that  
common denominator

$$= \frac{\cos^2 x}{\sin x}$$

Because  
 $\cos^2 x + \sin^2 x = 1$

$$= \cos x \frac{\cos x}{\sin x}$$

some algebra

$$= \cos x \cot x$$

$$\cot x = \frac{\cos x}{\sin x}$$

# Verifying Identities

Some things to note:

- ▶ Verifying an identity is **NOT** solving an equation.
- ▶ We do not "do the same thing" to both sides.
- ▶ We do not assume the statement is true. We **SHOW** it!
- ▶ Pick one side, and apply identities to it. The goal is to transform it to the other side.
- ▶ Usually try to work with the *most complicated* side. (It's usually easier to simplify a complicated expression than to complicate a simpler one!)
- ▶ Sometimes it helps to write everything in terms of sines and cosines—not always, but often.

Verify  $\tan\left(\frac{\pi}{2} - \beta\right) \tan(\beta) = 1$

We'll start with the left side.

$$\tan\left(\frac{\pi}{2} - \beta\right) \tan\beta = \cot\beta \tan\beta$$

$$= \frac{1}{\tan\beta} \tan\beta$$

$$= \frac{\tan\beta}{\tan\beta}$$

$$= 1$$

cofunction  
ID

$$\tan\left(\frac{\pi}{2} - \beta\right) = \cot\beta$$

reciprocal ID

$$\cot\beta = \frac{1}{\tan\beta}$$

Verify  $\frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}$

We'll start with the left side.

$$\frac{\sin x}{1-\cos x}$$

$$= \left( \frac{\sin x}{1-\cos x} \right) \left( \frac{1+\cos x}{1+\cos x} \right)$$

$$= \frac{\sin x (1+\cos x)}{1-\cos^2 x}$$

we want to use

$$\sin^2 x + \cos^2 x = 1$$

we will use

$$1-\cos^2 x = (1-\cos x)(1+\cos x)$$

$$= \frac{\sin x (1 + \cos x)}{\sin^2 x}$$

Pythagorean ID

$$= \frac{\sin x (1 + \cos x)}{\sin x \sin x}$$

$$= \frac{1 + \cos x}{\sin x}$$

cancel common factor