# August 13 Math 2306 sec. 53 Fall 2018

## Section 1: Concepts and Terminology

Suppose $y = \phi(x)$ is a differentiable function. We know that $dy/dx = \phi'(x)$ is another (related) function.

For example, if $y = \cos(2x)$, then $y$ is differentiable on $(-\infty, \infty)$. In fact,

$$\frac{dy}{dx} = -2\sin(2x).$$

Even $dy/dx$ is differentiable with $d^2y/dx^2 = -4\cos(2x)$.

# Suppose $y = \cos(2x)$

Note that $$\frac{d^2y}{dx^2} + 4y = 0.$$

We know $\dfrac{d^2y}{dx^2} = -4\cos(2x)$

so

$$\frac{d^2y}{dx^2} + 4y = -4\cos(2x) + 4\cos(2x) = 0$$

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# A *differential* equation

The equation

$$\frac{d^2y}{dx^2} + 4y = 0.$$

is an example of a **differential equation.**

**Questions:** If we only started with the equation, how could we determine that $\cos(2x)$ satisfies it? Also, is $\cos(2x)$ the only possible function that $y$ could be?

# Definition

A **Differential Equation** is an equation containing the derivative(s) of one or more dependent variables, with respect to one or more indendent variables.

**Solving** a differential equation refers to determining the dependent variable(s)—as function(s).

**Independent Variable:** will appear as one that derivatives are taken with respect to.

**Dependent Variable:** will appear as one that derivatives are taken of.

# Independent and Dependent Variables

Often, the derivatives indicate which variable is which:

$$\frac{dy}{dx} \qquad \frac{du}{dt} \qquad \frac{dx}{dr}$$

$y$ ← dependent, $x$ ← independent

$u$ ← dependent, $t$ ← independent

$x$ ← dependent, $r$ ← independent

$\Downarrow$

$u = f(t)$

u is a function

# Classifications

**Type:** An **ordinary differential equation (ODE)** has exactly one independent variable[1]. For example

$$\frac{dy}{dx} - y^2 = 3x, \quad \text{or} \quad \frac{dy}{dt} + 2\frac{dx}{dt} = t, \quad \text{or} \quad y'' + 4y = 0$$

A **partial differential equation (PDE)** has two or more independent variables. For example

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}, \quad \text{or} \quad \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

[1] These are the subject of this course.

# Classifications

**Order:** The order of a differential equation is the same as the highest order derivative appearing anywhere in the equation.

$$\frac{dy}{dx} - y^2 = 3x$$

1st order eqn.

$$y''' + (y')^4 = x^3$$

3rd order eqn.

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$$

2nd order equation.

# Notations and Symbols

We'll use standard derivative notations:

$$\text{Leibniz:}\quad \frac{dy}{dx},\quad \frac{d^2y}{dx^2},\quad \ldots\quad \frac{d^ny}{dx^n},\quad \text{or}\qquad \frac{d^4y}{dx^4} = y^{(4)}$$

$$\text{Prime \& superscripts:}\quad y',\quad y'',\quad \ldots\quad y^{(n)}.$$

Newton's **dot notation** may be used if the independent variable is time. For example if $s$ is a position function, then

$$\text{velocity is } \frac{ds}{dt} = \dot{s}, \quad \text{and acceleration is } \frac{d^2s}{dt^2} = \ddot{s}$$

# Notations and Symbols

An $n^{th}$ order ODE, with independent variable $x$ and dependent variable $y$ can always be expressed as an equation

$$F(x, y, y', \ldots, y^{(n)}) = 0$$

where $F$ is some real valued function of $n + 2$ variables.

**Normal Form:** If it is possible to isolate the highest derivative term, then we can write a **normal form** of the equation

$$\frac{d^n y}{dx^n} = f(x, y, y', \ldots, y^{(n-1)}).$$

$$\frac{d^2y}{dx^2} + 4y = 0$$ 2nd order

Has the form $F(x, y, y', y'') = 0$

where $F(x, y, y', y'') = y'' + 4y$

In normal form

$$\frac{d^2y}{dx^2} = -4y$$

looks like $\frac{d^2y}{dx^2} = f(x, y, y')$

where $f(x, y, y') = -4y$

# Notations and Symbols

If $n = 1$ or $n = 2$, an equation in normal form would look like

$$\frac{dy}{dx} = f(x, y) \quad \text{or} \quad \frac{d^2y}{dx^2} = f(x, y, y').$$

**Differential Form:** A first order equation may appear in the form

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

Differential forms may be written in normal form in a couple of ways.

If $N(x,y) \neq 0$

$$N(x,y)\,dy = -M(x,y)\,dx$$

Divide by $N(x,y)$ and $dx$ to get

$$\frac{dy}{dx} = \frac{-M(x,y)}{N(x,y)}$$

Or if $M(x,y) \neq 0$ we could write

$$\frac{dx}{dy} = \frac{-N(x,y)}{M(x,y)}$$