August 15 MATH 1113 sec. 51 Fall 2018

A Review of Lines (Overview of sections 1.3 & 1.4)

Definition: A line is the collection of points (x, y) in the plane that satisfy an equation of the form

$$Ax + By = C$$

for fixed real numbers A, B, and C with at least one of A or B nonzero.

Special Case: Vertical Line

If $B = 0$ $(A \neq 0)$, the above equation can be written as $x = K$ for fixed K. The collection of points (K, y) is a vertical line.

Non-vertical Lines and Slope

Definition: The **slope** of a line is a measure of the *rate of change* in the y-value as it depends on a change in the x-value between two points on a line. **The slope of any non-vertical line is constant.**

Slope Formula: Given a pair of distinct points (x_1, y_1), (x_2, y_2) on a non-vertical line, the slope m is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

- Note that this is a ratio of the vertical change (rise), often denoted Δy, to the horizontal change (run), often denoted Δx, between these points.
- Since there is no horizontal change between points on a vertical line, a vertical line does not have a slope (i.e. slope is not defined).

Question

A line contains the points $(1, 1)$, $(3, 4)$ and $(-1, -2)$. The slope m of this line is

(a) $m = -\frac{3}{2}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{3-1} = \frac{3}{2}$

(b) $m = \frac{2}{3}$

(c) $m = -\frac{2}{3}$

(d) $m = \frac{3}{2}$

(e) the answer depends on which points are used

Intercepts & Slope-Intercept form

If a line crosses one of the axes, the point at which it does is called an intercept. An x-intercept is a point $(x_0, 0)$ and a y-intercept is a point $(0, y_0)$.

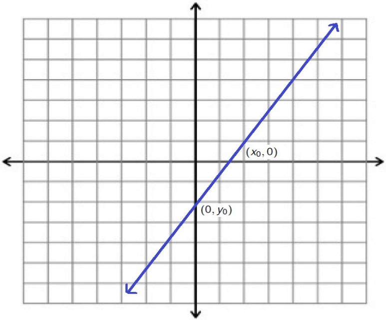

Figure: Note that at an intercept, the other coordinate value is zero.

Slope-Intercept Form

The equation of a non-vertical line can be written in the **slope-intercept** form

$$y = mx + b$$

where m is the slope and $(0, b)$ is the y-intercept.

We can contrast this with $Ax + By = C$ usually called *standard form* or the form

$$y - y_1 = m(x - x_1)$$

called *point-slope* form where m is the slope and (x_1, y_1) is some point on the line.

Parallel & Perpendicular

Definition: Two lines are **parallel** if the have the same slope (or if both are vertical).

Definition: Two lines with defined slopes m_1 and m_2 are **perpendicular** if $m_1 m_2 = -1$. In addition, a vertical line is perpendicular to any line of slope zero.

Question

Which of the following line(s) is/are perpendicular to the line $y = 4x - 3$?

(i) $y = 4x + 3$ (ii) $x + 4y = 3$ (iii) $y = -\frac{1}{4}x$ (iv) $4x + y = -3$

$y = \frac{-1}{4}x + \frac{3}{4}$ $y = -4x - 3$

(a) (ii) and (iii)

(b) (i) and (ii)

(c) (iii) only

(d) (ii) and (iv)

(e) (ii), (iii), and (iv)

Section 1.2: Relations & Functions

Given two sets of objects, we can define a **relation** on these sets.

Definition: A **relation** is a correspondence (or a mapping) between a first set, called the **domain**, and a second set, called the **range**, such that each element in the domain corresponds to (is related to/is mapped to) at least one element in the range.

Relations may be represented in several ways. The ones we'll use frequently are lists of ordered pairs, graphs, and equations.

Side Note: The range may be part of a larger set, called the codomain, not all of whose elements are included in the relation.

Relation Examples

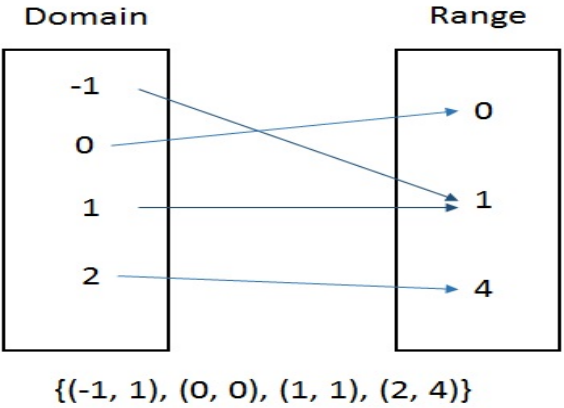

Figure: A relation on the sets $\{-1, 0, 1, 2\}$ and $\{0, 1, 4\}$ where the correspondence is that the range element is the square of the domain element. The relation is shown pictorially and as a set of ordered pairs. Note the order in the pairs is always (domain, range).

Relation Examples

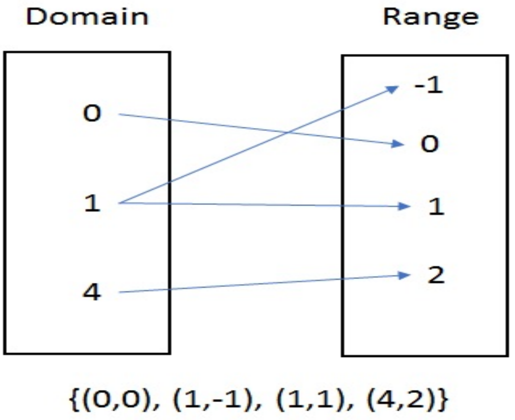

Figure: A relation on the sets $\{0, 1, 4\}$ and $\{-1, 0, 1, 2\}$ where the correspondence is that the range element is a number whose square is the domain element. The relation is shown pictorially and as a set of ordered pairs. Note the order in the pairs is always (domain, range).

Functions

These two examples highlight a characteristic of relations.

- Each domain element has at least one arrow emanating from it, and
- each range element has at least one arrow leading to it.

We have special names associated with restricting the number of such arrows. The most notable of these comes from insisting that each domain element have exactly one arrow coming from it.

Definition: A **function** on two sets is a relation in which each element of the domain corresponds to (is related to/is mapped to) exactly one element in the range.

Example

Function: The first relation $\{(-1,1),(0,0),(1,1),(2,4)\}$ is a function.
Note that no two ordered pairs have the same first element!

Not a Function: The second relation $\{(0,0),(1,-1),(1,1),(4,2)\}$ is NOT a function.
Note that two ordered pairs, $(1,-1)$ and $(1,1)$, have the same first element!

Observation: The definition of a function says that a domain element can only appear in one ordered pair. It does not restrict the appearance of a range element!

Question

Which (if any) of the following relations is a function?

(a) $\{(0,0),(1,1),(2,2)\}$

(b) $\{(0,1),(0,2),(0,3)\}$

(c) $\{(1,3),(-1,3),(7,7),(0,7)\}$

(d) (a) and (c)

(e) none of the above

Question

Consider the function $\{(1,3),(-1,3),(7,7),(0,7)\}$. Identify the domain and range of this function.

(a) the domain is $\{-1,1,0,7\}$ and the range is $\{3,7\}$

(b) the domain is $\{3,7\}$ and the range is $\{-1,1,0,7\}$

(c) the domain is $\{-1,1,0,3,7\}$ and the range is $\{3,7\}$

(d) the domain cannot be determined from this representation

Function Notation

Throughout this course and calculus, we are primarily interested in functions whose domain and range are subsets of the real numbers.

The correspondence (mapping) rule is often expressed by equations providing an algebraic formula telling us how to determine the range element to which a domain element is mapped.

A few preliminary remarks on Function Notation

- We will use a variable character to represent domain elements—usually x (but not always).
- We will use a variable character to represent corresponding range elements—usually y (again, we're not married to y).
- We assign a character name to our functions as well using specific notation—often we use f, sometime g, h or something else.
- The domain and range are not always stated explicitly, but we can often infer them.
- **Reading, writing, and using function notation properly is one of the most critical learning outcomes in this course.**

Function Notation: An example

Consider the equation $y = 3x - 4$. We know that this equation defines a line in the plane. That is, it defines a set of points

$$(x, y) = (x, 3x - 4)$$

where x and y are elements of the set of real[1] numbers $\mathbb{R}$.

The equation $y = 3x - 4$ defines a function. Let's call this function f. We can express this in **function notation** as

$$f(x) = 3x - 4.$$

In English, this reads as

f of *x* equals three *x* minus 4.

[1]The symbol $\mathbb{R}$ denotes the set of all real number.

Function Notation: An example

$$\text{Let} \quad f(x) = 3x - 4, \quad \text{and suppose} \quad y = f(x)$$

- In $f(x)$, f is the function and x is its **argument**.
- x represents an element of the domain, $f(x)$ is an element of the range.
- Since $y = f(x)$, x is called the **independent variable** and y is called the **dependent variable**.
- $y = f(x)$ reads "y equals f of x"
- The collection of points $(x, f(x))$, for each x in the domain, is called **the graph of f**.